Runtime Verification for Alternation-Free HyperLTL

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joint work with Noel Brett and Umair Siddique

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September 27, 2016

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S. Agrawal and B. Bonakdarpour. **Runtime Verification of k-safety Hyperproperties in HyperLTL** (CSF 2016).

Why Alternation-free HyperLTL?

Consider formula

$$\forall\pi.\exists\pi'.\varphi$$

To reason about such a formula, we should have all traces.

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This cannot be done by runtime techniques only.

Consider formula

$$\forall \pi . \forall \pi' . \varphi$$

One can detect violation of such a formula by discovering two traces that do not satisfy φ.

Presentation outline

1. Finite Semantics for LTL

2. Challenges in RV for HyperLTL

3. Rewriting-based RV Algorithm for Alternation-free HyperLTL Formulas
   - Identifying the Propositions of Interest
   - Rewriting-based RV for FLTL

4. Conclusion

Framework

Definitions

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The set of all finite (respectively, infinite) words are Σ^* (respectively, Σ^ω).

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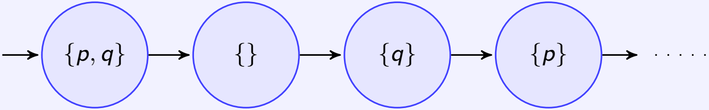
Example



Finite LTL (FLTL [Manna, Pnueli - 95)

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FLTL Syntax

The syntax of FLTL is identical to that of LTL and the semantics is based on the truth values $\mathbb{B}_2 = \{\bot, \top\}$.

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FLTL Semantics

The semantics of FLTL for atomic propositions and Boolean operators are identical to those of LTL.

Finite LTL

FLTL Semantics

Let φ, φ_1, and φ_2 be LTL formulas, and $u = u_0 u_1 \cdots u_n$ be a finite word.

$$[u \models_F \mathbf{X}\, \varphi] = \begin{cases} [u^1 \models_F \varphi] & \text{if} \quad u^1 \neq \epsilon \\ \bot & \text{otherwise} \end{cases}$$

$$[u \models_F \bar{\mathbf{X}}\, \varphi] = \begin{cases} [u^1 \models_F \varphi] & \text{if} \quad u^1 \neq \epsilon \\ \top & \text{otherwise} \end{cases}$$

$$[u \models_F \varphi_1 \,\mathbf{U}\, \varphi_2] = \begin{cases} \top & \text{if} \quad \exists k \in [0, n] : [u^k \models_F \varphi_2] = \top \;\wedge \\ & \qquad \forall l \in [0, k) : [u^l \models_F \varphi_1] = \top \\ \bot & \text{otherwise} \end{cases}$$

FLTL

Example

FLTL

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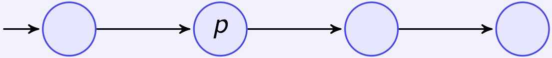
$[u \models_F \mathbf{X}p] = \top$



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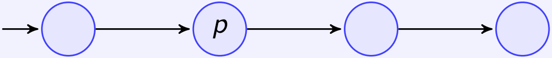

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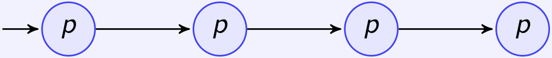

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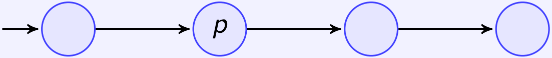

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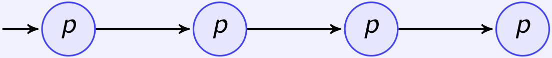

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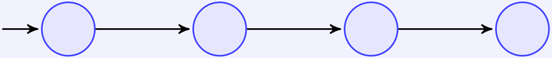

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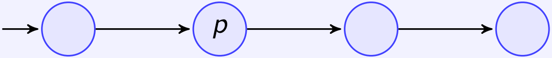

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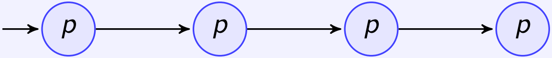

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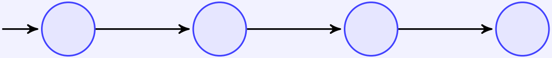
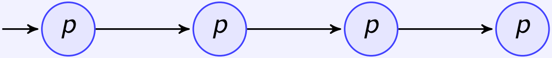


$[u \models_F \mathbf{G}p] = \top$



FLTL

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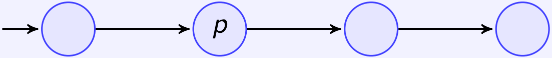

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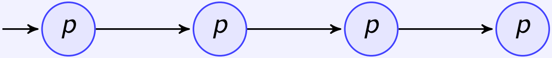

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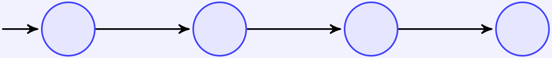

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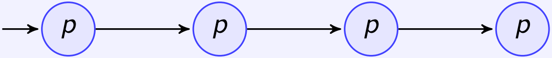

FLTL Put into Perspective

FLTL evaluates a property for a finite word regardless of future executions.

Monitorability

An LTL formula φ is monitorable iff

$$\forall u \in \Sigma^*. \exists u' \in \Sigma^*. [uu' \models_F \varphi] \in \{\bot, \top\}$$

Example

Formula $\mathbf{GF}p$ is not monitorable.

Formual $a\, \mathbf{U}\, b$ is monitorable.

Presentation outline

Finite Semantics for HyperLTL

For a finite trace t, let $t[i, j]$ denote the subtrace of t from position i up to and including position j. And, $t[i, ..]$ denotes $t[i, |t| - 1]$

Trace assignment function is now $\Pi : \mathcal{V} \to \Sigma^*$.

We define:

$$t[i,j] \;=\; \begin{cases} \epsilon & \text{if} \quad i > |t| \\ t[i, \min(j, |t| - 1)] & \text{otherwise} \end{cases}$$

Finite Semantics for HyperLTL

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$$\left[\Pi \models^{F}_{T} \mathbf{X}\, \varphi\right] = \begin{cases} [\Pi[1,..] \models^{F}_{T} \varphi] & \text{if} \quad \Pi[1,..] \neq \epsilon \\ \bot & \text{otherwise} \end{cases}$$

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$$\left[\Pi \models^{F}_{T} \varphi_1 \,\mathbf{U}\, \varphi_2\right] = \begin{cases} \top & \text{if} \quad \exists i \geq 0 : \Pi[i,..] \neq \epsilon \wedge [\Pi[i,..] \models^{F}_{T} \varphi_2] = \top \;\wedge \\ & \qquad \forall j \in [0,i) : [\Pi[j,..] \models^{F}_{T} \varphi_1] = \top \\ \bot & \text{otherwise} \end{cases}$$

Monitorability in HyperLTL

Trace Set Prefix Relation

Let U be a finite set of finite traces and V is a finite or infinite set of traces, then the prefix relation $U \leq V$ is defined as

$$U \leq V \;\equiv\; \forall u \in U.\, (\exists v \in V.\, u \leq v)$$

Note that V may contain traces that have no prefix in U.

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A HyperLTL formula φ is monitorable iff

$$\forall M \in \mathcal{P}^*(\Sigma^*). \exists M' \in \mathcal{P}^*(\Sigma^*).\, [MM' \models \varphi] \in \{\bot, \top\}$$

RV of HyperLTL: Challenges

- Hyperproperties are more complex than traditional trace properties; i.e., we need to reason over multiple execution traces.

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- Monitoring an execution may depend on the evaluation of past and future executions.

Goal is to develop RV algorithms for alternation-free HyperLTL formulas.

(a subset of k-safety hyperproperties)

Motivating Example 1

$$\phi = \forall \pi_1 . \forall \pi_2 . \; a_{\pi_1} \; \mathbf{U} \; b_{\pi_2}$$

$t_1 =$ a a a b

$t_2 =$ a a

Motivating Example 1

$$\phi = \forall \pi_1 . \forall \pi_2 .\ a_{\pi_1}\ \mathbf{U}\ b_{\pi_2}$$

$$t_1 = \text{a a a b}$$

$$t_2 = \text{a a}$$

- Traces π_1 and π_2 violate the formula (even t_2 does not individually satisfy ϕ.

Motivating Example 2

$$\phi = \forall \pi_1 . \forall \pi_2 .\ a_{\pi_1}\ \mathbf{U}\ b_{\pi_2}$$

$t_1 =$ a a a a a b

$t_2 =$ a a a a b b

$t_3 =$ a a b b b b

$t_4 =$ a a a b a a

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- Traces t_1, t_2, t_3, t_4, individually satisfy the formula ϕ.

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- Traces t_1, t_2, t_3, t_4, individually satisfy the formula ϕ.
- However, any combination of these traces violates the formula as satisfaction must happen at the same location in all traces.

We require the information about the index of satisfaction

Motivating Example 3

$$\phi = \forall \pi_1 . \forall \pi_2 . \ a_{\pi_1} \longrightarrow \mathbf{F}\, b_{\pi_2}$$

$t_1 =$ d c f

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- Traces t_1 and t_2, individually satisfy the formula ϕ.
- However, t_1 and t_2 collectively violate the formula as b never happened in π_1

Progressing through the traces requires information from previous traces

Motivating Example 4

$$\phi = \forall \pi_1 \forall \pi_2 .\ \mathbf{G}(a_{\pi_1} \longrightarrow a_{\pi_2})$$

$t_1 =$ a b b a b

$t_2 =$ a e c a c

$t_3 =$ a a c a a

Motivating Example 4

$$\phi = \forall \pi_1 \forall \pi_2.\ \mathbf{G}(a_{\pi_1} \longrightarrow a_{\pi_2})$$

$t_1 =$ a b b a b

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$t_2 =$ a e c a c

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- Traces t_1 and t_2 satisfy the formula ϕ.
- However, trace t_3 violates their agreement; i.e., satisfaction must happen at the same location in all traces

Trace quantification brings more expressiveness as compared to LTL

Monitoring HyperLTL: Idea

$$\phi = \forall \pi_1 . \forall \pi_2 .\ a_{\pi_1}\ \mathbf{U}\ b_{\pi_2}$$

$t_1 =$ a a a a a b

$$constraint = \mathbf{X}^{[0,4]} \neg b \wedge \mathbf{X}^5 b$$

$\mathbf{X}^{[i,k]}\psi$ represents $\mathbf{X}^i \psi \wedge \cdots \wedge \mathbf{X}^k \psi$

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Incoming traces have to satisfy ϕ and these constraints

$t_2 =$ a a a a b b ~~constraints~~

$t_3 =$ a a b b b b ~~constraints~~

$t_4 =$ a a a b a a ~~constraints~~

Monitoring HyperLTL: Idea (contd.)

$$\phi = \forall \pi_1 . \forall \pi_2 . \forall \pi_3 . \; (a_{\pi_1} \; \mathbf{U} \; b_{\pi_2}) \; \mathbf{U} \; c_{\pi_3}$$

$t_1 =$ ab a ac ac a b

Monitoring HyperLTL: Idea (contd.)

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$t_2 =$ a b ab a ac b

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$$\phi = \forall \pi_1 . \forall \pi_2 . \forall \pi_3 .\ (a_{\pi_1} \ \mathbf{U}\ b_{\pi_2})\ \mathbf{U}\ c_{\pi_3}$$

$t_1 =$ ab a ac ac a b

$t_2 =$ a b ab a ac b

$t_3 =$ b a ac a a ab

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$t_1 =$ ab | a | ac | ac | a | b

$t_2 =$ a | b | ab | a | ac | b

$t_3 =$ b | a | ac | a | a | ab

How to generate constraints in this case?

Semantics of U

$$\Pi \models_{\mathtt{T}} \phi_1\ \mathbf{U}\ \phi_2 \quad \text{iff} \quad \exists \mathtt{i} \geq 0.(\Pi[\mathtt{i},\infty] \models_{\mathtt{T}} \phi_2 \land \forall \mathtt{j}.0 \leq \mathtt{j} < \mathtt{i}.\Pi[\mathtt{j},\infty] \models_{\mathtt{T}} \phi_1)$$

- In this formula, the propositions of interest are b and c.
- The first satisfaction of c should be at the same index in each trace.
- Each satisfaction of b should be agreed upon in all traces.

Proposed Monitoring Approach

- Rewriting-based monitoring for HyperLTL
  - Using rewriting for the evaluation and progression of formula with respect to incoming events
  - Generating constraints to ensure the agreement among traces

- Hyper-Monitors for HyperLTL
  - Using LTL_4 (or RV-LTL) with counters to monitors the progress of each trace
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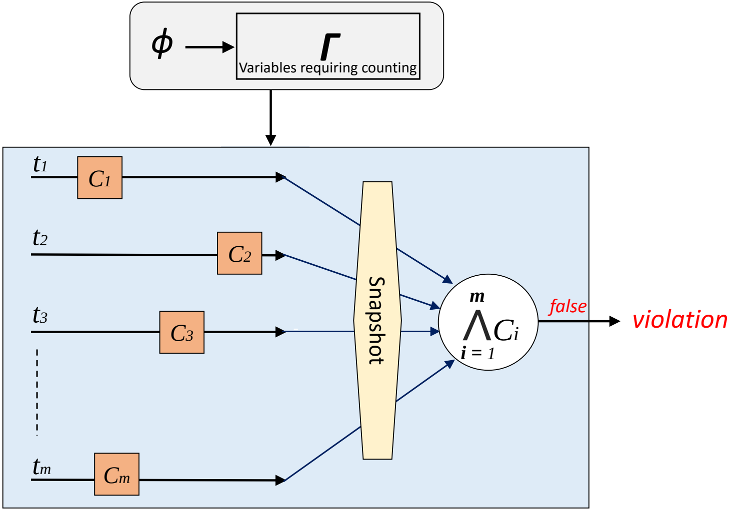
$$\phi = \forall \pi_1 . \forall \pi_2 . \forall \pi_3 . \ (a_{\pi_1} \ \mathbf{U} \ b_{\pi_2}) \ \mathbf{U} \ c_{\pi_3}$$

Presentation outline

Proposed Monitoring Algorithms: An Overview

Rewriting-based Monitoring for HyperLTL



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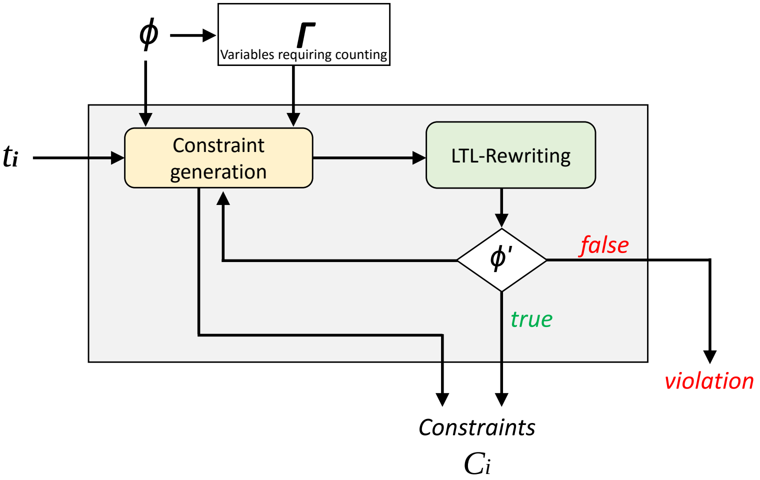


Outline

Algorithm to Find the set of Propositions of Interest (Γ)

Input: Syntax tree of the HyperLTL formula (ft)
Output: Set Γ
function $\Gamma(ft)$
$node := root(ft)$
$V := \{\}$ $\triangleright$*tracks record of trace quantifiers under the scope of "**U**"*
If $\neg(distinctQuantifiers(all.leaves))$ return $(\{\})$
match ($node$) **with**

Algorithm to Find the set of Propositions of Interest (Γ)

$|\ (\vee)$:

match $(\mathit{left}.\mathit{node})$ $(\mathit{right}.\mathit{node})$ **with**

$|\quad \mathit{leaf}_1 \qquad \mathit{leaf}_2\ :$

if $(\mathit{leaf}_1.\pi \neq \mathit{leaf}_2.\pi)$

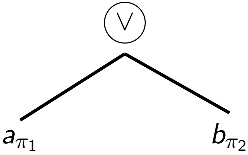
return $(\{\mathit{leaf}_1 \vee \mathit{leaf}_2\})$

else

return $(\{\})$

end if

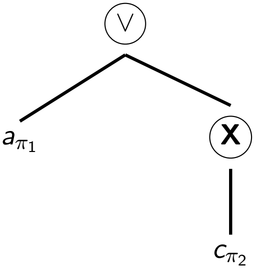
$\Gamma(a_{\pi_1} \vee b_{\pi_2}) = \{a \vee b\}$



Algorithm to Find the set of Propositions of Interest (Γ)

| ($\vee$):
match (*left.node*) (*right.node*) **with**
$\quad | \quad leaf_1 \quad - \quad : \text{return } (\{leaf_1\} \cup \Gamma(right.node))$

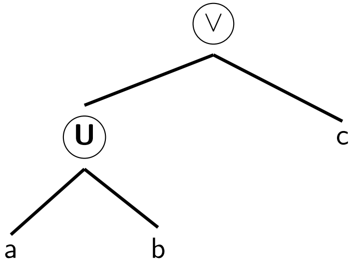
$\Gamma(a_{\pi_1} \vee \mathbf{X} c_{\pi_2}) = \{a\} \cup \Gamma(\mathbf{X} c_{\pi_2})$



Algorithm to Find the set of Propositions of Interest (Γ)

| ($\vee$):
  match (*left.node*) (*right.node*) **with**
    | $-$ *leaf*$_2$: return $(\Gamma(left.node) \cup \{leaf_2\})$

$\Gamma((a_{\pi_1}\ \mathbf{U}\ b_{\pi_2}) \vee c_{\pi_3}) =$
$\Gamma(a_{\pi_1}\ \mathbf{U}\ b_{\pi_2}) \cup \{c\}$



Algorithm to Find the set of Propositions of Interest (Γ)

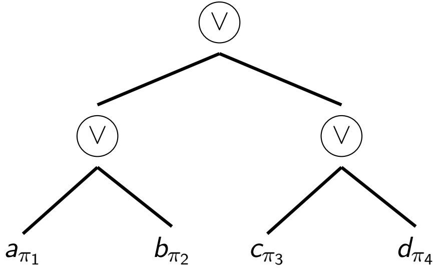
$\mid (\vee)$:

match $(left.node)$ $(right.node)$ **with**

$\mid \ \{\vee, \neg\} \quad \{\vee, \neg\}$: return $(\Gamma(left.node) \veebar \Gamma(right.node))$

$\veebar$ represents logical "OR" operation among the elements of two sets

$$\Gamma((a_{\pi_1} \vee b_{\pi_2}) \vee (c_{\pi_3} \vee d_{\pi_4})) =$$
$$\Gamma(a_{\pi_1} \vee b_{\pi_2}) \veebar \Gamma(c_{\pi_3} \vee d_{\pi_4}) =$$
$$\{a \vee b\} \veebar \{c \vee d\} =$$
$$\{a \vee b \vee c \vee d\}$$



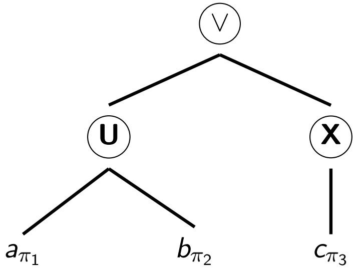
Algorithm to Find the set of Propositions of Interest (Γ)

| ($\vee$):

match (*left.node*) (*right.node*) **with**

| $-$ $-$: return ($\Gamma(\mathit{left.node}) \cup \Gamma(\mathit{right.node})$)

$\Gamma((a_{\pi_1}\ \mathbf{U}\ b_{\pi_2}) \vee \mathbf{X} c_{\pi_3}) =$
$\Gamma(a_{\pi_1}\ \mathbf{U}\ b_{\pi_2}) \cup \Gamma(\mathbf{X} c_{\pi_3})$



Algorithm to Find the set of Propositions of Interest (Γ)

| (**U**) :

match (*left.node*) (*right.node*) **with**

| $leaf_1$ $leaf_2$:

if $(V - \{leaf_2.\pi\} \neq \emptyset \vee leaf_1.\pi \neq leaf_2.\pi)$

return $(\{leaf_2\})$

else

return $(\{\})$

end if

Algorithm to Find the set of Propositions of Interest (Γ)

$|$ ($\mathbf{U}$) :

match ($left.node$) ($right.node$) **with**

$|$ $leaf_1$ $leaf_2$:

if $(V - \{leaf_2.\pi\} \neq \emptyset \vee leaf_1.\pi \neq leaf_2.\pi)$

return $(\{leaf_2\})$

else

return $(\{\})$

end if

$|$ $-$ $leaf_2$:

if $(\nexists x \in (left.node).x \in \{\mathbf{X}, \mathbf{U}\})$

return $(\# \odot \{leaf_2\})$

else

return $(\Gamma(left.node) \cup \# \odot \{leaf_2\})$

end if

$\#$ indicates that the corresponding variables need to be counted only once

$\odot$ represents application of unary operators (e.g., $\neg, \mathbf{X}$) to the elements of a set

Algorithm to Find the set of Propositions of Interest (Γ)

| (**U**) :
  match $(\mathit{left.node})\ (\mathit{right.node})$ **with**
    | leaf_1 $-$:
      $V \leftarrow V \cup \{\mathit{leaf}_1.\pi\}$;
      return $(\Gamma(\mathit{right.node}))$

Algorithm to Find the set of Propositions of Interest (Γ)

$|\ (\mathbf{U})$:

match $(left.node)\ (right.node)$ **with**

$|\quad leaf_1 \qquad - :$

$V \leftarrow V \cup \{leaf_1.\pi\}$;

return $(\Gamma(right.node))$

$|\quad - \qquad - :$

if $(\nexists x \in (left.node).x \in \{\mathbf{X}, \mathbf{U}\})$

return $(\# \odot \Gamma(right.node))$

else

return $(\Gamma(left.node) \cup \# \odot \Gamma(right.node))$

end if

Algorithm to Find the set of Propositions of Interest (Γ)

match (*node*) **with**

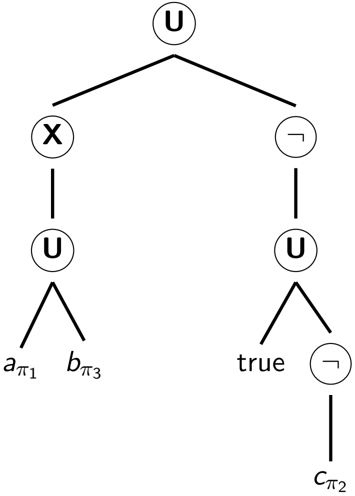
| (*leaf*) : return ($\{leaf\}$)

| ($\neg$) : return ($\neg \odot \Gamma(child.node)$)

| ($\mathbf{X}$) : return ($\mathbf{X} \odot \Gamma(child.node)$)

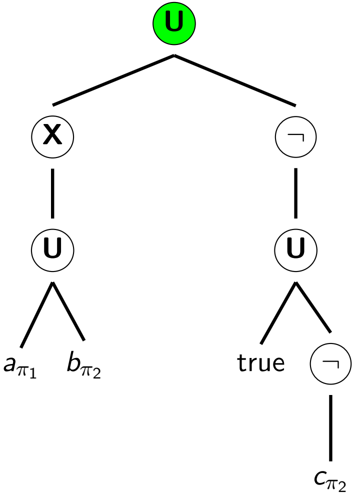
Algorithm to Find Γ: Running Example

$$\phi = \forall\pi_1.\forall\pi_2.\forall\pi_3 \mathbf{X}(a_{\pi_1} \,\mathbf{U}\, b_{\pi_3})\mathbf{U}\; \overbrace{\neg(\mathit{true}\,\mathbf{U}\,\neg c_{\pi_2})}^{\mathbf{G}c}$$



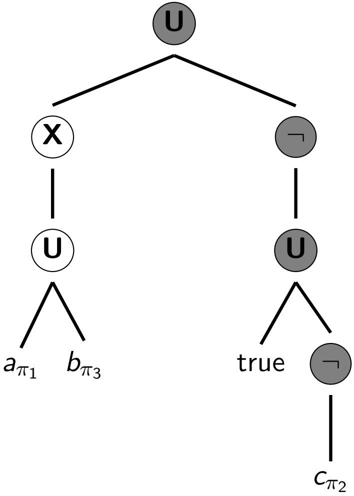
Algorithm to Find Γ: Running Example

$$\Gamma(\mathbf{X}(a_{\pi_1}\ \mathbf{U}\ b_{\pi_3})) \cup \Gamma(\neg(\mathit{true}\mathbf{U}\neg c_{\pi_2}))$$



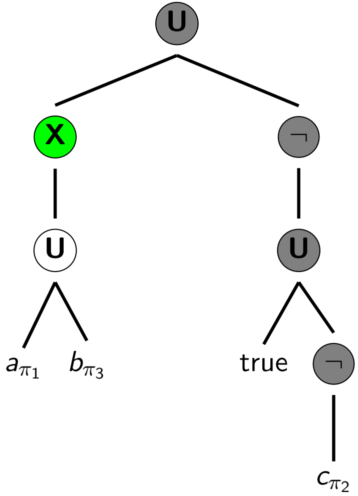
Algorithm to Find Γ: Running Example

$\Gamma(\mathbf{X}(a_{\pi_1} \ \mathbf{U} \ b_{\pi_3}))$



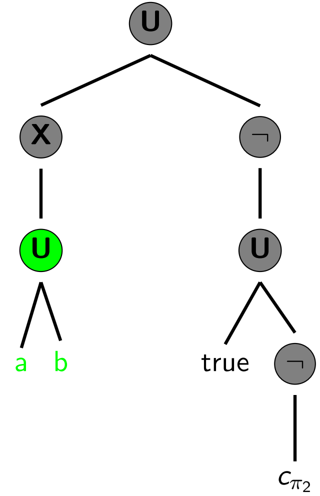
Algorithm to Find Γ: Running Example

$\mathbf{X} \odot \Gamma((a_{\pi_1} \ \mathbf{U} \ b_{\pi_3}))$



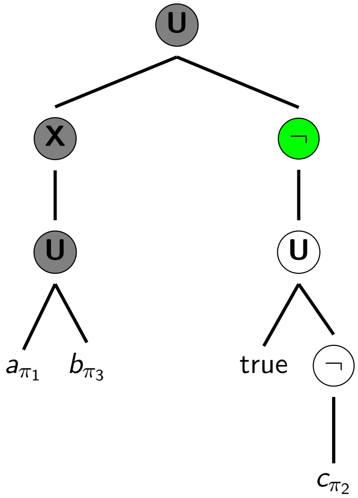
Algorithm to Find Γ: Running Example

$\mathbf{X} \odot \{b\} = \{\mathbf{X} b\}$



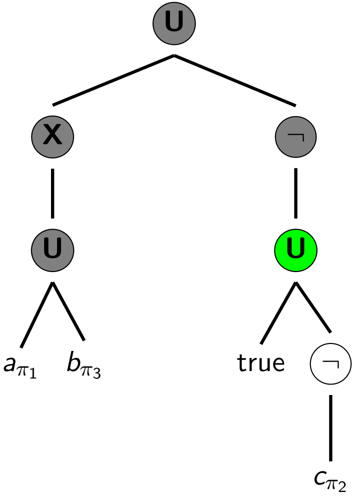
Algorithm to Find Γ: Running Example

$\neg \odot \Gamma((true\mathbf{U}\neg c_{\pi_2}))$



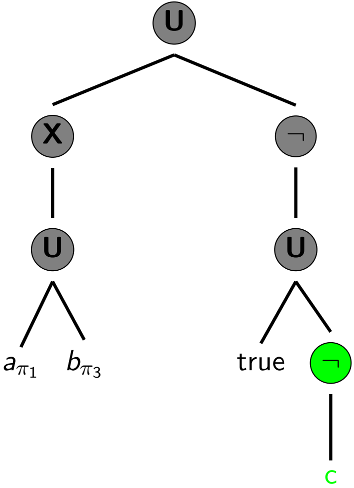
Algorithm to Find Γ: Running Example

$\neg \odot \Gamma(\neg c_{\pi_2})$



Algorithm to Find Γ: Running Example

$\neg \odot \{\neg c_{\pi_2}\} = \{c\}$

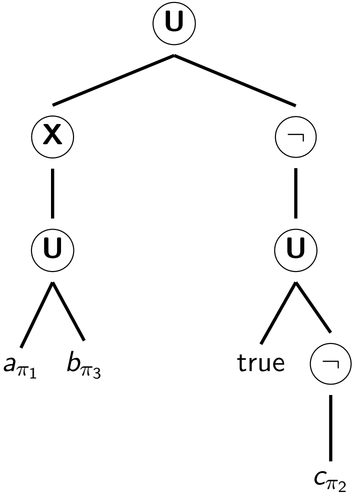


Algorithm to Find Γ: Running Example

$$\phi = \forall\pi_1.\forall\pi_2.\forall\pi_3\mathbf{X}(a_{\pi_1}\ \mathbf{U}\ b_{\pi_3})\mathbf{U}\ \overbrace{\neg(\mathit{true}\ \mathbf{U}\ \neg c_{\pi_2})}^{\mathbf{G}c}$$

$$\Gamma(\mathbf{X}(a_{\pi_1}\ \mathbf{U}\ b_{\pi_3})) \cup \Gamma(\neg(\mathit{true}\mathbf{U}\neg c_{\pi_2}))$$

$$\{\mathbf{X}b\} \cup \{c\} = \{\mathbf{X}b, c\}$$



Outline

Rewriting Algorithm [Havelund, Rosu, 2001]

REWRITE (ϕ, e)

Input: LTL formula ϕ, Event e
Output: Formula ψ

Rewriting Algorithm [Havelund, Rosu, 2001]

REWRITE (ϕ, e)

match (ϕ) **with**
$|\ (\phi_1 \vee \phi_2)$:
return $(REWRITE(\phi_1, e) \vee REWRITE(\phi_2, e))$

Rewriting Algorithm [Havelund, Rosu, 2001]

REWRITE (ϕ, e)

match (ϕ) **with**
$|\ (\phi_1 \vee \phi_2)$:
return $(REWRITE(\phi_1, e) \vee REWRITE(\phi_2, e))$

$\phi = (a \vee b)$ In-coming event $= a$

Rewriting Algorithm [Havelund, Rosu, 2001]

REWRITE (ϕ, e)

match (ϕ) **with**
$|\ (\phi_1 \vee \phi_2)$:
return $(REWRITE(\phi_1, e) \vee REWRITE(\phi_2, e))$

$\phi = (a \vee b)$ In-coming event $= a$
$\implies$ return $(REWRITE(a, a) \vee REWRITE(b, a))$

Rewriting Algorithm [Havelund, Rosu, 2001]

REWRITE (ϕ, e)

match (ϕ) **with**
$| \ (\phi_1 \vee \phi_2)$:
return $(REWRITE(\phi_1, e) \vee REWRITE(\phi_2, e))$

$\phi = (a \vee b)$ In-coming event $= a$
$\implies$ return $(REWRITE(a, a) \vee REWRITE(b, a))$

Further steps will return the satisfaction by finding $(a \vDash a) \vee (b \vDash a)$

Rewriting Algorithm [Havelund, Rosu, 2001]

REWRITE (ϕ, e)

$|\ (\phi_1 \ \mathbf{U}\ \phi_2)$:
if(last_event(e)) **then**
  return (REWRITE(ϕ_2, e))
else
  return (REWRITE(ϕ_2, e) $\vee$ *REWRITE*(ϕ_1, e) $\wedge$ ($\phi_1 \ \mathbf{U}\ \phi_2$)))

Rewriting Algorithm [Havelund, Rosu, 2001]

REWRITE (ϕ, e)

$\mid (\phi_1 \ \mathbf{U} \ \phi_2)$:
if(last_event(e)) **then**
  return (REWRITE(ϕ_2, e))
else
  return (REWRITE($\phi_2, e) \vee REWRITE(\phi_1, e) \wedge (\phi_1 \ \mathbf{U} \ \phi_2)$))

$\phi = (a \ \mathbf{U} \ b)$ In-coming event $= a$

Rewriting Algorithm [Havelund, Rosu, 2001]

REWRITE (ϕ, e)

$| \ (\phi_1 \ \mathbf{U} \ \phi_2)$:
if(last_event(e)) **then**
return (REWRITE(ϕ_2, e))
else
return (REWRITE(ϕ_2, e) $\vee$ $\mathit{REWRITE}(\phi_1, e) \wedge (\phi_1 \ \mathbf{U} \ \phi_2)$))

$\phi = (a \ \mathbf{U} \ b)$ In-coming event $= a$
$\implies$ return $(\mathit{REWRITE}(b, a) \vee (\mathit{REWRITE}(a, a) \wedge (a \ \mathbf{U} \ b)))$

Rewriting Algorithm [Havelund, Rosu, 2001]

REWRITE (ϕ, e)

$\mid (\phi_1 \ \mathbf{U} \ \phi_2)$:
if(last_event(e)) **then**
  return (REWRITE(ϕ_2, e))
else
  return (REWRITE(ϕ_2, e) $\vee$ $REWRITE(\phi_1, e) \wedge (\phi_1 \ \mathbf{U} \ \phi_2)$))

$\phi = (a \ \mathbf{U} \ b)$ In-coming event $= a$
$\implies$ return $(\mathit{False} \vee (\mathit{True} \wedge (a \ \mathbf{U} \ b)))$

Rewriting Algorithm [Havelund, Rosu, 2001]

REWRITE (ϕ, e)

$| \ (\phi_1 \ \mathbf{U} \ \phi_2)$:
if(last_event(e)) **then**
  return (REWRITE(ϕ_2, e))
else
  return (REWRITE($\phi_2, e) \vee \mathit{REWRITE}(\phi_1, e) \wedge (\phi_1 \ \mathbf{U} \ \phi_2)$))

$\phi = (a \ \mathbf{U} \ b)$ In-coming event $= a$
$\implies$ return $(\mathit{False} \vee (\mathit{True} \wedge (a \ \mathbf{U} \ b)))$

Formula has not yet been satisfied/violated

$a \ \mathbf{U} \ b$ will be used again to find the satisfaction/violation

Rewriting Algorithm [Havelund, Rosu, 2001]

REWRITE (ϕ, e)

$|$ $(\mathbf{X}\phi)$:
if(last_event(e)) **then**
  return (*False*)
else
  return REWRITE(ϕ, e)

Rewriting Algorithm [Havelund, Rosu, 2001]

REWRITE (ϕ, e)

$\mid (\mathbf{X}\phi)$:
if(last_event(e)) **then**
  return ($False$)
else
  return REWRITE(ϕ, e)

$\phi = (\mathbf{X}b)$ In-coming event $= a$

Rewriting Algorithm [Havelund, Rosu, 2001]

REWRITE (ϕ, e)

$\mid (\mathbf{X}\phi)$:
if(last_event(e)) **then**
    return (*False*)
else
    return REWRITE(ϕ, e)

$\phi = (\mathbf{X} b)$ In-coming event $= a$
$\Longrightarrow$ return (REWRITE(b, e))

Rewriting Algorithm [Havelund, Rosu, 2001]

REWRITE (ϕ, e)

$\mid$ $(\mathbf{X}\phi)$:
if(last_event(e)) **then**
  return ($False$)
else
  return REWRITE(ϕ, e)

$\phi = (\mathbf{X}b)$ In-coming event $= a$
$\implies$ return (REWRITE(b, e))

The next in-coming event will have to satisfy b

Rewriting Algorithm [Havelund, Rosu, 2001]

REWRITE (ϕ, e)

$| \; (\mathbf{X}\phi)$:
if(last_event(e)) **then**
    return (*False*)
else
    return REWRITE(ϕ, e)

$\phi = (\mathbf{X}b)$ In-coming event $= a$
$\Longrightarrow$ return (REWRITE(b, e))

The next in-coming event will have to satisfy b

If a was last event, then violation found.

Implementation of "**strong next**"

Rewriting Algorithm [Havelund, Rosu, 2001]

REWRITE (ϕ, e)

$\mid (\phi[e])$:
if$(e \models \phi)$ **then**
    return $(True)$
elseif$(e \not\models \phi)$
    return$(False)$
$\mid (True)$:   return $(True)$
$\mid (False)$:   return $(False)$

Rewriting Algorithm [Havelund, Rosu, 2001]

REWRITE (ϕ, e)

match (ϕ) **with**
| $(\phi_1 \vee \phi_2)$:
  return $(REWRITE(\phi_1, e) \vee REWRITE(\phi_2, e))$
| $(\phi_1 \ \mathbf{U} \ \phi_2)$:
if(last_event(e)) **then**
  return (REWRITE(ϕ_2, e))
else
  return (REWRITE$(\phi_2, e) \vee REWRITE(\phi_1, e) \wedge (\phi_1 \ \mathbf{U} \ \phi_2)$))
| $(\mathbf{X}\phi)$:
if(last_event(e)) **then**
  return $(False)$
else
  return REWRITE(ϕ, e)
| $(\phi[e])$:
if$(e \models \phi)$ **then**
  return $(True)$
elseif$(e \not\models \phi)$
  return$(False)$
| $(True)$: return $(True)$
| $(False)$: return $(False)$

Rewriting Algorithm - Example

$$\boxed{\phi = a \ \mathbf{U} \ b \quad \pi = \epsilon}$$

Rewriting Algorithm - Example

$\boxed{\phi = a \ \mathbf{U} \ b \quad \pi = a}$ – Event comes in

Rewriting Algorithm - Example

$$\boxed{\phi = a \; \mathbf{U} \; b \quad \pi = a}$$

$$\phi \leftarrow \text{REWRITE}(\phi, a)$$

Rewriting Algorithm - Example

$$\boxed{\phi = a \ \mathbf{U} \ b \quad \pi = a}$$

$$\phi \leftarrow \text{REWRITE}(\phi, a)$$
$$\implies \text{ return } (REWRITE(b, a) \vee (REWRITE(a, a) \wedge (a \ \mathbf{U} \ b)))$$

Rewriting Algorithm - Example

$$\boxed{\phi = a \ \mathbf{U} \ b \quad \pi = a}$$

$$\phi \leftarrow \text{REWRITE}(\phi, a)$$

$$\implies \text{return } (\mathit{False} \vee (\mathit{True} \wedge (a \ \mathbf{U} \ b)))$$

Rewriting Algorithm - Example

$$\boxed{\phi = a \ \mathbf{U} \ b \quad \pi = a}$$

$$\phi \leftarrow \text{REWRITE}(\phi, a)$$
$$\phi \leftarrow (a \ \mathbf{U} \ b)$$

Rewriting Algorithm - Example

$\boxed{\phi = a \ \mathbf{U} \ b \quad \pi = aa}$ – Event comes in

Rewriting Algorithm - Example

$$\boxed{\phi = a\ \mathbf{U}\ b \quad \pi = aa}$$

$$\phi \leftarrow \text{REWRITE}(\phi, a)$$

Rewriting Algorithm - Example

$$\boxed{\phi = a \; \mathbf{U} \; b \quad \pi = aa}$$

$$\phi \leftarrow \text{REWRITE}(\phi, a)$$

$$\Longrightarrow \;\; \text{return } (REWRITE(b, a) \vee (REWRITE(a, a) \wedge (a \; \mathbf{U} \; b)))$$

Rewriting Algorithm - Example

$$\boxed{\phi = a\ \mathbf{U}\ b \quad \pi = a\color{red}{a}}$$

$$\phi \leftarrow \text{REWRITE}(\phi, a)$$

$$\Longrightarrow \ \text{return } (\color{red}{False} \vee (\color{red}{True} \wedge (a\ \mathbf{U}\ b)))$$

Rewriting Algorithm - Example

$$\boxed{\phi = a \; \mathbf{U} \; b \quad \pi = a{\color{red}a}}$$

$$\phi \leftarrow \text{REWRITE}(\phi, a)$$
$$\phi \leftarrow (a \; \mathbf{U} \; b)$$

Rewriting Algorithm - Example

$\boxed{\phi = a \ \mathbf{U} \ b \quad \pi = aab}$ – Event comes in

Rewriting Algorithm - Example

$$\boxed{\phi = a \ \mathbf{U} \ b \quad \pi = aab}$$

$$\phi \leftarrow \text{REWRITE}(\phi, b)$$

Rewriting Algorithm - Example

$$\boxed{\phi = a \ \mathbf{U} \ b \quad \pi = aab}$$

$$\phi \leftarrow \text{REWRITE}(\phi, b)$$

$$\implies \text{return } (REWRITE(b, b) \vee (REWRITE(a, b) \wedge (a \ \mathbf{U} \ b)))$$

Rewriting Algorithm - Example

$$\boxed{\phi = a \ \mathbf{U} \ b \quad \pi = aab}$$

$$\phi \leftarrow \text{REWRITE}(\phi, b)$$

$$\implies \text{return } (\mathit{True} \vee (\mathit{False} \wedge a \ \mathbf{U} \ b))$$

Rewriting Algorithm - Example

$$\boxed{\phi = a \ \mathbf{U} \ b \quad \pi = aab}$$

$$\phi \leftarrow \text{REWRITE}(\phi, b)$$
$$\phi \leftarrow True$$

Rewriting Algorithm - Example

$$\boxed{\phi = a \ \mathbf{U} \ b \quad \pi = aa\textcolor{red}{b}}$$

$$\phi \leftarrow \text{REWRITE}(\phi, b)$$

$$\phi \leftarrow \mathit{True}$$

Formula is satisfied!

Rewriting-based Monitoring for HyperLTL: Challenges

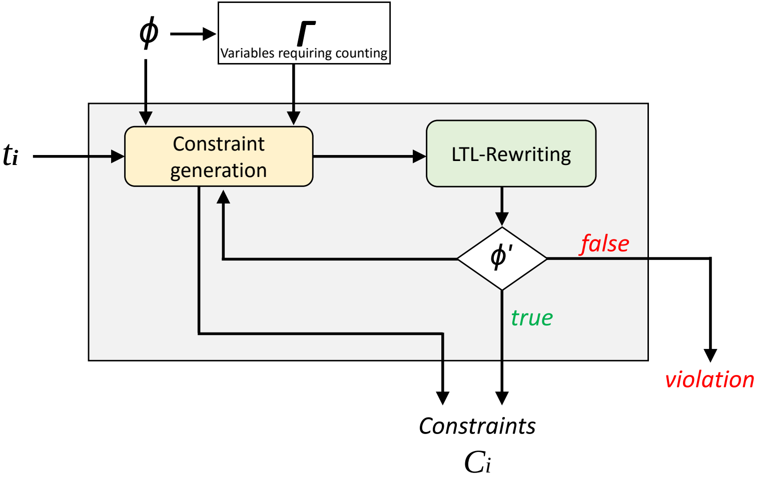
- Progressing through the traces requires information from previous traces
  $\phi = \forall\pi_1.\forall\pi_2.\ a_{\pi_1} \longrightarrow \mathbf{F}\, b_{\pi_2}$
- Tracking the satisfaction of sub-formulas is required
- Constraints generation is dynamic depending upon the agreement amongst traces

Rewriting-based Monitoring: Proposed Approach

- Find the set of variables which require counting (Γ)
- Use rewriting to check the satisfaction and progress of the formula
- Generate the constraints (required for trace agreement) for each incoming event

These steps are performed for all incoming traces

Rewriting-based Monitoring for HyperLTL



Rewriting-based Monitoring

Input: HyperLTL formula ϕ, Γ, set of incoming traces M
Output: $\lambda = \{\bot, ?\}$

Rewriting-based Monitoring

Input: HyperLTL formula ϕ, Γ, set of incoming traces M
Output: $\lambda = \{\bot, ?\}$

```
while (1) do
    for each m ∈ M do
        C_m ← ConstraintsHandler (φ, Γ)        ▷ Generate constraints
```

Rewriting-based Monitoring

Input: HyperLTL formula ϕ, Γ, set of incoming traces M
Output: $\lambda = \{\bot, ?\}$

while (1) **do**
  for each $m \in M$ **do**
    $C_m \leftarrow$ `ConstraintsHandler` (ϕ, Γ) ▷ *Generate constraints*
Take a snapshot for counters $\mathcal{C} = \{C_1, C_2, \cdots, C_m\}$ at time instant
$\beta = \mathtt{SAT}(\bigwedge_{m=1}^{M}(C_m))$ ▷ *Check the satisfiability of constraints*
if $(\beta = \mathit{false})$ **then**
  $\lambda := \bot$
return (λ)

Rewriting-based Monitoring

```
ConstraintsHandler (\phi, \Gamma)
if (\phi = \phi_1 \vee \phi_2) then
    \psi_1 \leftarrow ConstraintsHandler (\phi_1, \Gamma(\phi_1))
    \psi_2 \leftarrow ConstraintsHandler (\phi_2, \Gamma(\phi_2))
    if (\psi_1 = False \wedge \psi_2 = False) then
        return (False)
    else if (\psi_1 = False) then
        return (\psi_2)
    else if (\psi_2 = False) then
        return (\psi_1)
        else
            return (\psi_1 \wedge \psi_2)
else
    if (\phi := \phi_1 U \phi_2 \wedge ((\phi_1 := \phi_L \vee \phi_R) \wedge
    distinctQuantifiers(\phi_L, \phi_R) )) then
        \psi_1 \leftarrow ConstraintsHandler (\phi_L U \phi_2, \Gamma)
        \psi_2 \leftarrow ConstraintsHandler (\phi_R U \phi_2, \Gamma)
        if \psi_1 = False \wedge \psi_2 = False then
            return (False)
        else if \psi_1 = False then
            return ((\phi_R U \phi_2) \wedge \psi_2)
        else if \psi_2 = False then
            return ((\phi_L U \phi_2) \wedge \psi_1)
        else
            return (((\phi_R U \phi_2) \vee (\phi_R U \phi_2)) \wedge \psi_1)
    else
        r \leftarrow ConstraintsTraces(\Gamma,\phi)
        if (r = False) then
            return False
        else
            return r
```

Rewriting-based Monitoring

ConstraintsHandler (ϕ, Γ)
if $(\phi = \phi_1 \vee \phi_2)$ **then**
  $\psi_1 \leftarrow$ ConstraintsHandler $(\phi_1, \Gamma(\phi_1))$
  $\psi_2 \leftarrow$ ConstraintsHandler $(\phi_2, \Gamma(\phi_2))$
  if $(\psi_1 = \mathit{False} \wedge \psi_2 = \mathit{False})$ **then**
    return (False) ▷ *Return False if both formulas are False*
  else if $(\psi_1 = \mathit{False})$ **then**
    return (ψ_2)
  else if $(\psi_2 = \mathit{False})$ **then**
    return (ψ_1)
  else
    return $(\psi_1 \wedge \psi_2)$

- Divide ϕ into subformulas
- Seperate variables from Γ

- Constraints refinement

Rewriting-based Monitoring

ConstraintsHandler (ϕ, Γ)
else
 if $(\phi := \phi_1 \ \mathbf{U}\ \phi_2 \wedge ((\phi_1 := \phi_L \vee \phi_R) \wedge$ distinctQuantifiers(ϕ_L, ϕ_R) $))$ **then**
  $\psi_1 \leftarrow$ ConstraintsHandler $(\phi_L\ \mathbf{U}\ \phi_2, \Gamma)$
  $\psi_2 \leftarrow$ ConstraintsHandler $(\phi_R\ \mathbf{U}\ \phi_2, \Gamma)$
  if $(\psi_1 = False \wedge \psi_2 = False)$ **then**
   return $(False)$
  else if $(\psi_1 = False)$ **then**
   return $((\phi_R\ \mathbf{U}\ \phi_2) \wedge \psi_2)$
  else if $(\psi_2 = False)$ **then**
   return $((\phi_L\ \mathbf{U}\ \phi_2) \wedge \psi_1)$
  else
   return $(((\phi_L\ \mathbf{U}\ \phi_2) \vee (\phi_R\ \mathbf{U}\ \phi_2)) \wedge \psi_1)$
 else
  $r \leftarrow$ ConstraintsTraces(Γ, ϕ)
  if $(r = False)$ **then**
   return $False$
  else
   return r

- Left side of Until is disjunction with different quantifiers
- Divide to find which side of disjunction finds satisfaction

- Constraints are refined according to the satisfaction of disjunction

- Generate Constraint for each trace

Rewriting-based Monitoring - Algorithm for Constraints

ConstraintsTraces(Γ: *Set of variables require counting,* ϕ)
$\Gamma' \leftarrow \Gamma$
for *each* $a \in \Gamma$ **do**
    $i_a \leftarrow 0$
$i \leftarrow 0$
$r \leftarrow True$
$\phi' \leftarrow$ quantifier-elimination(ϕ)   ▷*Eliminate trace quantifiers*
while getEvent *(e)* **do**
    $\phi' \leftarrow$ REWRITE (e, ϕ')
    if $(\phi' = False)$ **then**
        return ϕ'
    for *(each* $a \in \Gamma$ *s.t.* $e \models a$*)* **do**
        if $(a = a'_{\#})$ **then**
            $\Gamma \leftarrow \Gamma \setminus a$
        $r \leftarrow r \wedge \mathbf{X}^i a$
        if $a \in \Gamma'$ **then**
            $\Gamma' \leftarrow \Gamma' \setminus a$
        if $(i > 0) \wedge (i_a \neq i)$ **then**
            $r \leftarrow r \wedge \mathbf{X}^{[i_a, i-1]} \neg a$
        $i_a \leftarrow i + 1$
    if $(\phi' = True)$ **then**
        Break
    for *(each* $a \in \Gamma$ *s.t.* $a \ = \mathbf{X}a'$*)* **do**
        $\Gamma \leftarrow (\Gamma \setminus \{a\}) \cup \{a'\}$
        $\Gamma' \leftarrow (\Gamma' \setminus \{a\}) \cup \{a'\}$
        $i_a{+}{+}$
    $i{+}{+}$
for *each* $b \in \Gamma'$ **do**
    $r \leftarrow r \wedge \mathbf{G} \neg b$
return r

Rewriting-based Monitoring - Algorithm for Constraints

ConstraintsTraces(Γ: *Set of variables require counting*, ϕ)
$\Gamma' \leftarrow \Gamma$
for *each* $a \in \Gamma$ **do**
  $i_a \leftarrow 0$
$i \leftarrow 0$
$r \leftarrow True$

- Set counter and state variables here
- i is the main counter for incoming events
- r represents constraints

Rewriting-based Monitoring - Algorithm for Constraints

ConstraintsTraces(Γ: *Set of variables require counting*, ϕ)
$\Gamma' \leftarrow \Gamma$
for *each* $a \in \Gamma$ **do**
  $i_a \leftarrow 0$
$i \leftarrow 0$
$r \leftarrow True$
$\phi' \leftarrow$ quantifier-elimination(ϕ)   ▷*Eliminate trace quantifiers*
while getEvent *(e)* **do**
  $\phi' \leftarrow$ REWRITE (e, ϕ')
  if $(\phi' = False)$
  then
    return ϕ'

- Set counter and state variables here
- i is the main counter for incoming events
- r represents constraints

- *Events of a trace come in*
- *Use rewriting for formula evaluation and progression*
- *In case of violation, False is returned*

Rewriting-based Monitoring - Algorithm for Constraints

ConstraintsTraces(Γ: *Set of variables require counting*, ϕ)

$\Gamma' \leftarrow \Gamma$
for *each* $a \in \Gamma$ **do**
  $i_a \leftarrow 0$
$i \leftarrow 0$
$r \leftarrow$ *True*

- Set counter and state variables here
- i is the main counter for incoming events
- r represents constraints

$\phi' \leftarrow$ quantifier-elimination(ϕ) ▷*Eliminate trace quantifiers*

while getEvent *(e)* **do**
  $\phi' \leftarrow$ REWRITE (e, ϕ')
  if $(\phi' = False)$
  then
    return ϕ'

- *Events of a trace come in*
- *Use rewriting for formula evaluation and progression*
- *In case of violation, False is returned*

  for *(each* $a \in \Gamma$ *s.t.* $e \vDash a$*)* **do**
    if $(a = a'_{\#})$ **then**
      $\Gamma \leftarrow \Gamma \setminus a$
    $r \leftarrow r \wedge \mathbf{X}^i a$
    if $a \in \Gamma'$ **then**
      $\Gamma' \leftarrow \Gamma' \setminus a$
    if $(i > 0) \wedge (i_a \neq i)$ **then**
      $r \leftarrow r \wedge \mathbf{X}^{[i_a, i-1]} \neg a$
    $i_a \leftarrow i + 1$

-Check if incoming event needs to be counted once
- Delete such variable from Γ
- Generate the constraint ensuring its occurance at location i
- Generate the constraints ensuring its absence from other lolcations
- Update the event counter

Rewriting-based Monitoring - Algorithm for Constraints

`ConstraintsTraces`(Γ: *Set of variables require counting*, ϕ)
$\Gamma' \leftarrow \Gamma$
for *each* $a \in \Gamma$ **do**
    $i_a \leftarrow 0$
$i \leftarrow 0$
$r \leftarrow$ *True*

- Set counter and state variables here
- i is the main counter for incoming events
- r represents constraints

$\phi' \leftarrow$ `quantifier-elimination`(ϕ)   ▷*Eliminate trace quantifiers*

while `getEvent` *(e)* **do**
    $\phi' \leftarrow$ `REWRITE` (e, ϕ')
    if $(\phi' = \mathit{False})$
    then
        return ϕ'

- *Events of a trace come in*
- *Use rewriting for formula evaluation and progression*
- *In case of violation, False is returned*

    for *(each* $a \in \Gamma$ *s.t.* $e \models a$*)* **do**
        if $(a = a'_{\#})$ **then**
            $\Gamma \leftarrow \Gamma \setminus a$
        $r \leftarrow r \wedge \mathbf{X}^{i} a$
        if $a \in \Gamma'$ **then**
            $\Gamma' \leftarrow \Gamma' \setminus a$
        if $(i > 0) \wedge (i_a \neq i)$ **then**
            $r \leftarrow r \wedge \mathbf{X}^{[i_a, i-1]} \neg a$
        $i_a \leftarrow i + 1$

-Check if incoming event needs to be counted once
- Delete such variable from Γ
- Generate the constraint ensuring its occurance at location i
- Generate the constraints ensuring its absence from other lolcations
- Update the event counter

    if $(\phi' = \mathit{True})$ **then**
        `Break`

If formula is true then stop generating constraints

Rewriting-based Monitoring - Algorithm for Constraints

ConstraintsTraces(Γ: *Set of variables require counting*, ϕ)
$\Gamma' \leftarrow \Gamma$
for *each* $a \in \Gamma$ **do**
  $i_a \leftarrow 0$
$i \leftarrow 0$
$r \leftarrow True$

- Set counter and state variables here
- i is the main counter for incoming events
- r represents constraints

$\phi' \leftarrow$ quantifier-elimination(ϕ) ▷*Eliminate trace quantifiers*
while getEvent (e) **do**
  $\phi' \leftarrow$ REWRITE (e, ϕ')
  if $(\phi' = False)$
  then
    return ϕ'

- *Events of a trace come in*
- *Use rewriting for formula evaluation and progression*
- *In case of violation, False is returned*

  for (*each* $a \in \Gamma$ *s.t.* $e \models a$) **do**
    if $(a = a'_{\#})$ **then**
      $\Gamma \leftarrow \Gamma \setminus a$
    $r \leftarrow r \wedge \mathbf{X}^i a$
    if $a \in \Gamma'$ **then**
      $\Gamma' \leftarrow \Gamma' \setminus a$
    if $(i > 0) \wedge (i_a \neq i)$ **then**
      $r \leftarrow r \wedge \mathbf{X}^{[i_a, i-1]} \neg a$
    $i_a \leftarrow i + 1$

-Check if incoming event needs to be counted once
- Delete such variable from Γ
- Generate the constraint ensuring its occurance at location i
- Generate the constraints ensuring its absence from other locations
- Update the event counter

  if $(\phi' = True)$ **then**
    Break

If formula is true then stop generating constraints

  for (*each* $a \in \Gamma$ *s.t.* $a = \mathbf{X}a'$) **do**
    $\Gamma \leftarrow (\Gamma \setminus \{a\}) \cup \{a'\}$
    $\Gamma' \leftarrow (\Gamma' \setminus \{a\}) \cup \{a'\}$
    $i_a{+}{+}$
  $i{+}{+}$

-Counters that include "**X**" will have "**X**" removed
- To be counted next round

for *each* $b \in \Gamma'$ **do**
  $r \leftarrow r \wedge \mathbf{G}\neg b$
return r

Rewriting-Based Monitoring: Running Example 1

$$\phi = \forall \pi_1 . \forall \pi_2 .\ a_{\pi_1} \longrightarrow \mathbf{F} b_{\pi_2}$$

Step 1: Finding Γ

$\phi = \forall_{\pi_1 \pi_2} .\ \neg a_{\pi_1} \vee \mathbf{F} b_{\pi_2}$

$\phi = \forall_{\pi_1 \pi_2} .\ \neg a_{\pi_1} \vee (true \mathbf{U} b_{\pi_2})$

$\Gamma = \{\neg a\}$

Rewriting-Based Monitoring: Running Example 1

$$\phi = \forall \pi_1 . \forall \pi_2 .\ a_{\pi_1} \longrightarrow \mathbf{F} b_{\pi_2}$$

Step 1: Finding Γ

$\phi = \forall_{\pi_1 \pi_2}.\ \neg a_{\pi_1} \vee \mathbf{F} b_{\pi_2}$

$\phi = \forall_{\pi_1 \pi_2}.\ \neg a_{\pi_1} \vee (true \mathbf{U} b_{\pi_2})$

$\Gamma = \{\neg a\}$

Step 2: Separating formula and finding constraints for each part

$\pi_1 =$ d c b

$constraints_{\mathbf{F} b} =$

$constraints_{\neg a} =$

Rewriting-Based Monitoring: Running Example 1

$$\phi = \forall \pi_1 . \forall \pi_2 .\ a_{\pi_1} \longrightarrow \mathbf{F} b_{\pi_2}$$

Step 1: Finding Γ

$\phi = \forall_{\pi_1 \pi_2} .\ \neg a_{\pi_1} \vee \mathbf{F} b_{\pi_2}$

$\phi = \forall_{\pi_1 \pi_2} .\ \neg a_{\pi_1} \vee (true \mathbf{U} b_{\pi_2})$

$\Gamma = \{\neg a\}$

Step 2: Separating formula and finding constraints for each part

$\pi_1 =$ d c b

$constraints_{\mathbf{F} b} = ?$

$constraints_{\neg a} = \neg a$

Rewriting-Based Monitoring: Running Example 1

$$\phi = \forall \pi_1 . \forall \pi_2 .\ a_{\pi_1} \longrightarrow \mathbf{F} b_{\pi_2}$$

Step 1: Finding Γ

$\phi = \forall_{\pi_1 \pi_2} .\ \neg a_{\pi_1} \vee \mathbf{F} b_{\pi_2}$

$\phi = \forall_{\pi_1 \pi_2} .\ \neg a_{\pi_1} \vee (\mathit{true} \mathbf{U} b_{\pi_2})$

$\Gamma = \{\neg a\}$

Step 2: Separating formula and finding constraints for each part

$\pi_1 =$ d c b

$\mathit{constraints}_{\mathbf{F} b} = ?$

$\mathit{constraints}_{\neg a} = \neg a$

Rewriting-Based Monitoring: Running Example 1

$$\phi = \forall \pi_1 . \forall \pi_2 .\ a_{\pi_1} \longrightarrow \mathbf{F} b_{\pi_2}$$

Step 1: Finding Γ

$\phi = \forall_{\pi_1 \pi_2} .\ \neg a_{\pi_1} \vee \mathbf{F} b_{\pi_2}$

$\phi = \forall_{\pi_1 \pi_2} .\ \neg a_{\pi_1} \vee (true \mathbf{U} b_{\pi_2})$

$\Gamma = \{\neg a\}$

Step 2: Separating formula and finding constraints for each part

$\pi_1 =$ d c b

$constraints_{\mathbf{F} b} = \mathbf{F} b$

$constraints_{\neg a} = \neg a$

Rewriting-Based Monitoring: Running Example 1

$$\phi = \forall\pi_1.\forall\pi_2.\ a_{\pi_1} \longrightarrow \mathbf{F}\, b_{\pi_2}$$

Step 1: Finding Γ

$\phi = \forall_{\pi_1\pi_2}.\ \neg a_{\pi_1} \vee \mathbf{F}\, b_{\pi_2}$

$\phi = \forall_{\pi_1\pi_2}.\ \neg a_{\pi_1} \vee (\mathit{true}\,\mathbf{U}\, b_{\pi_2})$

$\Gamma = \{\neg a\}$

$\mathit{constraints}_{\mathbf{F}b} = \mathbf{F}\, b$

$\mathit{constraints}_{\neg a} = \neg a$

Step 3: Check incoming traces agree with the constraints

$\pi_2 =$ e e e

Rewriting-Based Monitoring: Running Example 1

$$\phi = \forall\pi_1.\forall\pi_2.\ a_{\pi_1} \longrightarrow \mathbf{F}\, b_{\pi_2}$$

Step 1: Finding Γ

$\phi = \forall_{\pi_1\pi_2}.\ \neg a_{\pi_1} \vee \mathbf{F}\, b_{\pi_2}$

$\phi = \forall_{\pi_1\pi_2}.\ \neg a_{\pi_1} \vee (true\mathbf{U}\, b_{\pi_2})$

$\Gamma = \{\neg a\}$

$constraints_{\mathbf{F}b} = \mathbf{F}\, b$

$constraints_{\neg a} = \neg a$

Step 3: Check incoming traces agree with the constraints

$\pi_2 =$ e e e

Rewriting-Based Monitoring: Running Example 1

$$\phi = \forall \pi_1 . \forall \pi_2 .\ a_{\pi_1} \longrightarrow \mathbf{F}\, b_{\pi_2}$$

Step 1: Finding Γ

$\phi = \forall_{\pi_1 \pi_2} .\ \neg a_{\pi_1} \vee \mathbf{F}\, b_{\pi_2}$

$\phi = \forall_{\pi_1 \pi_2} .\ \neg a_{\pi_1} \vee (\mathit{true}\, \mathbf{U}\, b_{\pi_2})$

$\Gamma = \{\neg a\}$

$\mathit{constraints}_{\mathbf{F} b} = \mathbf{F}\, b$

$\mathit{constraints}_{\neg a} = \neg a$

Step 3: Check incoming traces agree with the constraints

$\pi_2 =$ e e e

Rewriting-Based Monitoring: Running Example 1

$$\phi = \forall \pi_1 . \forall \pi_2 .\ a_{\pi_1} \longrightarrow \mathbf{F} b_{\pi_2}$$

Step 1: Finding Γ

$\phi = \forall_{\pi_1 \pi_2} .\ \neg a_{\pi_1} \vee \mathbf{F} b_{\pi_2}$

$\phi = \forall_{\pi_1 \pi_2} .\ \neg a_{\pi_1} \vee (\mathit{true} \mathbf{U} b_{\pi_2})$

$\Gamma = \{\neg a\}$

$\cancel{\mathit{constraints}_{\mathbf{F} b} = \mathbf{F} b}$

$\mathit{constraints}_{\neg a} = \neg a$

Step 3: Check incoming traces agree with the constraints

$\pi_2 =$ e e e

Rewriting-Based Monitoring: Running Example 1

$$\phi = \forall \pi_1.\forall \pi_2.\ a_{\pi_1} \longrightarrow \mathbf{F}\, b_{\pi_2}$$

Step 1: Finding Γ

$\phi = \forall_{\pi_1 \pi_2}.\ \neg a_{\pi_1} \vee \mathbf{F}\, b_{\pi_2}$
$\phi = \forall_{\pi_1 \pi_2}.\ \neg a_{\pi_1} \vee (\mathit{true}\, \mathbf{U}\, b_{\pi_2})$
$\Gamma = \{\neg a\}$

$\cancel{\mathit{constraints}_{\mathbf{F}b} = \mathbf{F}b}$
$\mathit{constraints}_{\neg a} = \neg a$

Step 3: Check incoming traces agree with the constraints

$\pi_2 =$ e e e

π_2 only agrees with $\mathit{constraints}_{\neg a}$, no b was found

Rewriting-Based Monitoring: Running Example 1

$$\phi = \forall \pi_1 . \forall \pi_2 . \; a_{\pi_1} \longrightarrow \mathbf{F} b_{\pi_2}$$

Step 1: Finding Γ

$\phi = \forall_{\pi_1 \pi_2} . \; \neg a_{\pi_1} \vee \mathbf{F} b_{\pi_2}$

$\phi = \forall_{\pi_1 \pi_2} . \; \neg a_{\pi_1} \vee (true \mathbf{U} b_{\pi_2})$

$\Gamma = \{\neg a\}$

$\cancel{constraints_{\mathbf{F} b} = \mathbf{F} b}$

$constraints_{\neg a} = \neg a$

Step 3: Check incoming traces agree with the constraints

$\pi_3 =$ a c b

Rewriting-Based Monitoring: Running Example 1

$$\phi = \forall \pi_1 . \forall \pi_2 .\ a_{\pi_1} \longrightarrow \mathbf{F}\, b_{\pi_2}$$

Step 1: Finding Γ

$\phi = \forall_{\pi_1 \pi_2}.\ \neg a_{\pi_1} \vee \mathbf{F}\, b_{\pi_2}$

$\phi = \forall_{\pi_1 \pi_2}.\ \neg a_{\pi_1} \vee (\mathit{true}\, \mathbf{U}\, b_{\pi_2})$

$\Gamma = \{\neg a\}$

$\cancel{\mathit{constraints}_{\mathbf{F}b} = \mathbf{F} b}$

$\cancel{\mathit{constraints}_{\neg a} = \neg a}$

Step 3: Check incoming traces agree with the constraints

$\pi_3 =$ a c b

π_3 disagrees with the last constraint, a was observed in the first location

Rewriting-Based Monitoring: Running Example 2

$$\phi = \forall_{\pi_1 \pi_2}.\ (a_{\pi_1}\ \mathbf{U}\ b_{\pi_2})\ \mathbf{U}\ c_{\pi_1}$$

Step 1: Finding Γ
$\Gamma = \{b, \#c\}$

Rewriting-Based Monitoring: Running Example 2

$$\phi = \forall_{\pi_1 \pi_2}.\ (a_{\pi_1}\ \mathbf{U}\ b_{\pi_2})\ \mathbf{U}\ c_{\pi_1}$$

$\Gamma = \{b, \#c\}$

Step 2: Finding constraints

$\pi_1 =$ a a ab ac ac ab

constraints $=$

Rewriting-Based Monitoring: Running Example 2

$$\phi = \forall_{\pi_1 \pi_2}.\ (a_{\pi_1}\ \mathbf{U}\ b_{\pi_2})\ \mathbf{U}\ c_{\pi_1}$$

$\Gamma = \{b, \#c\}$

Step 2: Finding constraints

$\pi_1 =$ a a ab ac ac ab

constraints $= \neg(bc) \wedge \mathbf{X}\neg(bc) \wedge \mathbf{X}^2 b \wedge \mathbf{X}^2 \neg c$

Rewriting-Based Monitoring: Running Example 2

$$\phi = \forall_{\pi_1 \pi_2}.\ (a_{\pi_1}\ \mathbf{U}\ b_{\pi_2})\ \mathbf{U}\ c_{\pi_1}$$

$\Gamma = \{b\}$

Step 2: Finding constraints

$\pi_1 =$ a a ab ac ac ab

$constraints = \neg(bc) \wedge \mathbf{X}\neg(bc) \wedge \mathbf{X}^2 b \wedge \mathbf{X}^2 \neg c \wedge \mathbf{X}^3 \neg b \wedge \mathbf{X}^3 c$

Rewriting-Based Monitoring: Running Example 2

$$\phi = \forall_{\pi_1\pi_2}.\ (a_{\pi_1}\ \mathbf{U}\ b_{\pi_2})\ \mathbf{U}\ c_{\pi_1}$$

$\Gamma = \{b\}$

Step 2: Finding constraints

$\pi_1 =$ a a ab ac ac ab

$constraints = \neg(bc) \wedge \mathbf{X}\neg(bc) \wedge \mathbf{X}^2 b \wedge \mathbf{X}^2 \neg c \wedge \mathbf{X}^3 \neg b \wedge \mathbf{X}^3 c \wedge \mathbf{X}^4 \neg b$

Rewriting-Based Monitoring: Running Example 2

$$\phi = \forall_{\pi_1 \pi_2}.\ (a_{\pi_1}\ \mathbf{U}\ b_{\pi_2})\ \mathbf{U}\ c_{\pi_1}$$

$\Gamma = \{b\}$

Step 2: Finding constraints

$\pi_1 =$ a a ab ac ac ab

$constraints = \neg(bc) \wedge \mathbf{X}\neg(bc) \wedge \mathbf{X}^2 b \wedge \mathbf{X}^2 \neg c \wedge \mathbf{X}^3 \neg b \wedge \mathbf{X}^3 c \wedge \mathbf{X}^4 \neg b \wedge \mathbf{X}^5 b$

Rewriting-Based Monitoring: Running Example 2

$$\phi = \forall_{\pi_1 \pi_2}.\ (a_{\pi_1}\ \mathbf{U}\ b_{\pi_2})\ \mathbf{U}\ c_{\pi_1}$$

$\Gamma = \{b\}$

Step 2: Finding constraints

$\pi_1 =$ a a ab ac ac ab

constraints $= \neg(bc) \wedge \mathbf{X}\neg(bc) \wedge \mathbf{X}^2 b \wedge \mathbf{X}^2 \neg c \wedge \mathbf{X}^3 \neg b \wedge \mathbf{X}^3 c \wedge \mathbf{X}^4 \neg b \wedge \mathbf{X}^5 b$

we count only the first occurrence of c

Rewriting-Based Monitoring: Running Example 2

$$\phi = \forall_{\pi_1\pi_2}.\ (a_{\pi_1}\ \mathbf{U}\ b_{\pi_2})\ \mathbf{U}\ c_{\pi_1}$$

$constraints = \neg(bc) \wedge \mathbf{X}\neg(bc) \wedge \mathbf{X}^2 b \wedge \mathbf{X}^2\neg c \wedge \mathbf{X}^3\neg b \wedge \mathbf{X}^3 c \wedge \mathbf{X}^4\neg b \wedge \mathbf{X}^5 b$

Step 3: Check incoming traces agree with the constraints

$\pi_2 =$ a a ab ac ac ab

Rewriting-Based Monitoring: Running Example 2

$$\phi = \forall_{\pi_1\pi_2}.\ (a_{\pi_1}\ \mathbf{U}\ b_{\pi_2})\ \mathbf{U}\ c_{\pi_1}$$

$constraints = \neg(bc) \wedge \mathbf{X}\neg(bc) \wedge \mathbf{X}^2 b \wedge \mathbf{X}^2\neg c \wedge \mathbf{X}^3\neg b \wedge \mathbf{X}^3 c \wedge \mathbf{X}^4\neg b \wedge \mathbf{X}^5 b$

Step 3: Check incoming traces agree with the constraints

$\pi_2 =$ a a ab ac ac ab

Rewriting-Based Monitoring: Running Example 2

$$\phi = \forall_{\pi_1\pi_2}.\ (a_{\pi_1}\ \mathbf{U}\ b_{\pi_2})\ \mathbf{U}\ c_{\pi_1}$$

$constraints = \neg(bc) \wedge \mathbf{X}\neg(bc) \wedge \mathbf{X}^2 b \wedge \mathbf{X}^2\neg c \wedge \mathbf{X}^3\neg b \wedge \mathbf{X}^3 c \wedge \mathbf{X}^4\neg b \wedge \mathbf{X}^5 b$

Step 3: Check incoming traces agree with the constraints

$\pi_2 =$ a a ab ac ac ab

Rewriting-Based Monitoring: Running Example 2

$$\phi = \forall_{\pi_1 \pi_2}.\ (a_{\pi_1}\ \mathbf{U}\ b_{\pi_2})\ \mathbf{U}\ c_{\pi_1}$$

$constraints = \neg(bc) \wedge \mathbf{X}\neg(bc) \wedge \mathbf{X}^2 b \wedge \mathbf{X}^2 \neg c \wedge \mathbf{X}^3 \neg b \wedge \mathbf{X}^3 c \wedge \mathbf{X}^4 \neg b \wedge \mathbf{X}^5 b$

Step 3: Check incoming traces agree with the constraints

$\pi_2 =$ a a ab ac ac ab

Rewriting-Based Monitoring: Running Example 2

$$\phi = \forall_{\pi_1\pi_2}.\ (a_{\pi_1}\ \mathbf{U}\ b_{\pi_2})\ \mathbf{U}\ c_{\pi_1}$$

$constraints = \neg(bc) \wedge \mathbf{X}\neg(bc) \wedge \mathbf{X}^2 b \wedge \mathbf{X}^2\neg c \wedge \mathbf{X}^3\neg b \wedge \mathbf{X}^3 c \wedge \mathbf{X}^4\neg b \wedge \mathbf{X}^5 b$

Step 3: Check incoming traces agree with the constraints

$\pi_2 =$ a a ab ac ac ab

Rewriting-Based Monitoring: Running Example 2

$$\phi = \forall_{\pi_1 \pi_2}. \; (a_{\pi_1} \; \mathbf{U} \; b_{\pi_2}) \; \mathbf{U} \; c_{\pi_1}$$

$constraints = \neg(bc) \wedge \mathbf{X}\neg(bc) \wedge \mathbf{X}^2 b \wedge \mathbf{X}^2 \neg c \wedge \mathbf{X}^3 \neg b \wedge \mathbf{X}^3 c \wedge \mathbf{X}^4 \neg b \wedge \mathbf{X}^5 b$

Step 3: Check incoming traces agree with the constraints

$\pi_2 =$ a a ab ac ac ab

π_2 does not create a violation, even though c is observed more than once

Rewriting-Based Monitoring: Running Example 2

$$\phi = \forall_{\pi_1\pi_2}.\ (a_{\pi_1}\ \mathbf{U}\ b_{\pi_2})\ \mathbf{U}\ c_{\pi_1}$$

$$\mathit{constraints} = \neg(bc) \wedge \mathbf{X}\neg(bc) \wedge \mathbf{X}^2 b \wedge \mathbf{X}^2\neg c \wedge \mathbf{X}^3\neg b \wedge \mathbf{X}^3 c \wedge \mathbf{X}^4\neg b \wedge \mathbf{X}^5 b$$

Step 3: Check incoming traces agree with the constraints

$\pi_3 =$ abc ab ab ac ac ab

Rewriting-Based Monitoring: Running Example 2

$$\phi = \forall_{\pi_1\pi_2}.\ (a_{\pi_1}\ \mathbf{U}\ b_{\pi_2})\ \mathbf{U}\ c_{\pi_1}$$

~~$constraints = \neg(bc) \wedge \mathbf{X}\neg(bc) \wedge \mathbf{X}^2 b \wedge \mathbf{X}^2\neg c \wedge \mathbf{X}^3\neg b \wedge \mathbf{X}^3 c \wedge \mathbf{X}^4\neg b \wedge \mathbf{X}^5 b$~~

Step 3: Check incoming traces agree with the constraints

$\pi_3 =$ abc ab ab ac ac ab

π_3 violates our constraint as the first location has b and c

Rewriting-Based Monitoring: Running Example 3

$$\phi = \forall_{\pi_1\pi_2}.\ (a_{\pi_1}\ \mathbf{U}\ b_{\pi_2})\ \mathbf{U}\ c_{\pi_1}$$

$$\Gamma = \{b, \#c\}$$

Rewriting-Based Monitoring: Running Example 3

$$\phi = \forall_{\pi_1 \pi_2}.\ (a_{\pi_1}\ \mathbf{U}\ b_{\pi_2})\ \mathbf{U}\ c_{\pi_1}$$

$$\Gamma = \{b, \#c\}$$

$\pi_1 =$ a a ab

$\pi_2 =$ a a

$constraints_{\pi_1} = \neg(bc) \wedge \mathbf{X}\neg(bc) \wedge \mathbf{X}^2 b \wedge \mathbf{X}^2 \neg c$

$constraints_{\pi_2} = \neg(bc) \wedge \mathbf{X}\neg(bc)$

Rewriting-Based Monitoring: Running Example 3

$$\phi = \forall_{\pi_1\pi_2}.\ (a_{\pi_1}\ \mathbf{U}\ b_{\pi_2})\ \mathbf{U}\ c_{\pi_1}$$

$$\Gamma = \{b, \#c\}$$

$\pi_1 =$ a a ab

$\pi_2 =$ a a

$constraints_{\pi_1} = \neg(bc) \wedge \mathbf{X}\neg(bc) \wedge \mathbf{X}^2 b \wedge \mathbf{X}^2\neg c$

$constraints_{\pi_2} = \neg(bc) \wedge \mathbf{X}\neg(bc)$

At this point, there is no constraint disagreement

Rewriting-Based Monitoring: Running Example 3

$$\phi = \forall_{\pi_1\pi_2}.\ (a_{\pi_1}\ \mathbf{U}\ b_{\pi_2})\ \mathbf{U}\ c_{\pi_1}$$

$$\Gamma = \{b, \#c\}$$

$\pi_1 =$ a a ab ac a ab

$\pi_2 =$ a a ab ab

$constraints_{\pi_1} = \neg(bc) \wedge \mathbf{X}\neg(bc) \wedge \mathbf{X}^2 b \wedge \mathbf{X}^2 \neg c \wedge \mathbf{X}^3 \neg b \wedge \mathbf{X}^3 c \wedge \mathbf{X}^4 \neg b \wedge \mathbf{X}^5 b$

$constraints_{\pi_2} = \neg(bc) \wedge \mathbf{X}\neg(bc) \wedge \mathbf{X}^2 b \wedge \mathbf{X}^2 \neg c \wedge \mathbf{X}^3 b \wedge \mathbf{X}^3 \neg c$

Rewriting-Based Monitoring: Running Example 3

$$\phi = \forall_{\pi_1\pi_2}.\ (a_{\pi_1}\ \mathbf{U}\ b_{\pi_2})\ \mathbf{U}\ c_{\pi_1}$$

$$\Gamma = \{b, \#c\}$$

$\pi_1 =$ a a ab ac a ab

$\pi_2 =$ a a ab ab

$$constraints_{\pi_1} = \neg(bc) \wedge \mathbf{X}\neg(bc) \wedge \mathbf{X}^2 b \wedge \mathbf{X}^2 \neg c \wedge \mathbf{X}^3 \neg b \wedge \mathbf{X}^3 c \wedge \mathbf{X}^4 \neg b \wedge \mathbf{X}^5 b$$

$$constraints_{\pi_2} = \neg(bc) \wedge \mathbf{X}\neg(bc) \wedge \mathbf{X}^2 b \wedge \mathbf{X}^2 \neg c \wedge \mathbf{X}^3 b \wedge \mathbf{X}^3 \neg c$$

There is a violation among constraints for π_1 and π_2

Presentation outline

1. Finite Semantics for LTL
2. Challenges in RV for HyperLTL
3. Rewriting-based RV Algorithm for Alternation-free HyperLTL Formulas
   - Identifying the Propositions of Interest
   - Rewriting-based RV for FLTL
4. Conclusion

Conclusion

Summary

- Monitorability for HyperLTL
- Finite semantics for HyperLTL
- Rewriting-based RV algorithm for alternation-free HyperLTL

Future Work

- Automata-based monitoring (hyper monitors!)
- RV for Alternating HyperLTL

Questions

Thank You!