HyperLTL

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based on joint work with Michael R. Clarkson,
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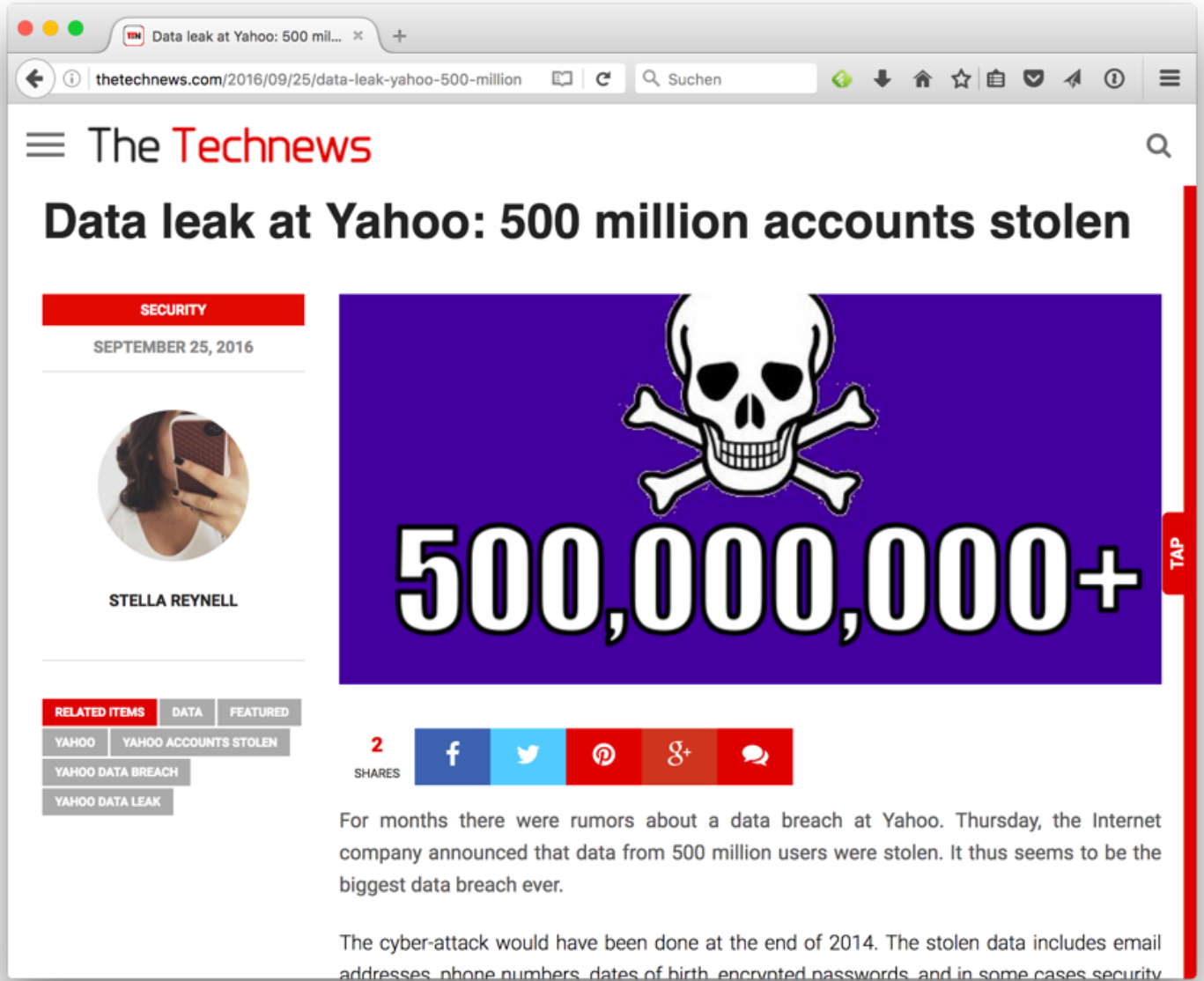
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Data leak at Yahoo: 500 million accounts stolen
SECURITY
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YAHOO
YAHOO ACCOUNTS STOLEN
YAHOO DATA BREACH
YAHOO DATA LEAK
2
SHARES
For months there were rumors about a data breach at Yahoo. Thursday, the Internet company announced that data from 500 million users were stolen. It thus seems to be the biggest data breach ever.
The cyber-attack would have been done at the end of 2014. The stolen data includes email addresses, phone numbers, dates of birth, encrypted passwords, and in some cases security

Major Incidents in Information Security

- Heartbleed **4.5m patient records leaked**

```
if (1 + 2 + payload + 16 > s->s3->rrec.length)
    return 0;
```

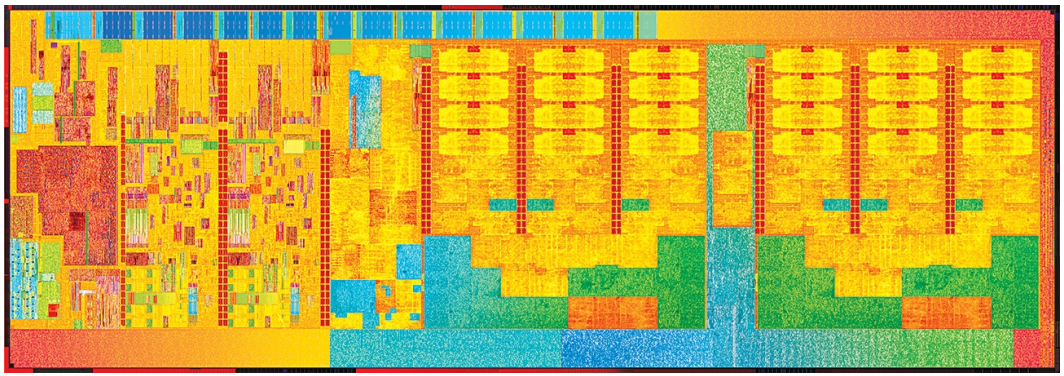
- Goto Fail **encryption of >300M devices broken**

```
if ((err = SSLHashSHA1.update(&hashCtx, &signedParams)) != 0)
   goto fail;
   goto fail;
```

- Shellshock **web servers attackable for 22 years**

```
parse_and_execute (temp_string, name, SEVAL_NONINT|SEVAL_NOHIST);
```

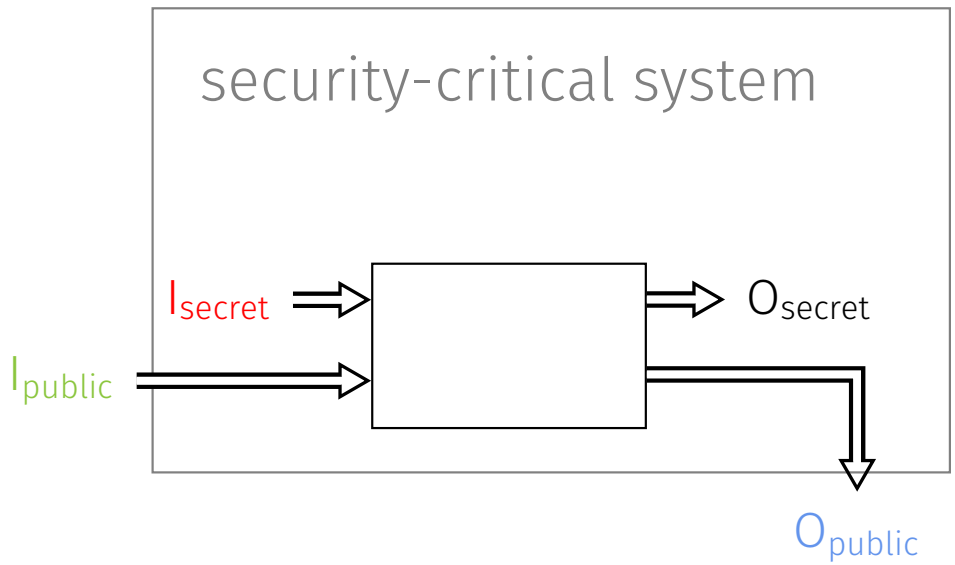
Embedded Systems / Hardware Security



Programming errors happen frequently

- Hard to fix once shipped
- “Specification Updates”

Information-flow control



Public output should only depend on public input.

Typical information-flow property: Noninterference

$$\forall t, t' \in \mathit{Traces}(K) : \; t =_{I_{public}} t' \;\Rightarrow\; t =_{O_{public}} t'$$

Hyperproperties

Clarkson&Schneider'10:

Hyperproperty H: a set of sets of traces

System K satisfies H iff $Traces(K) \in H$.

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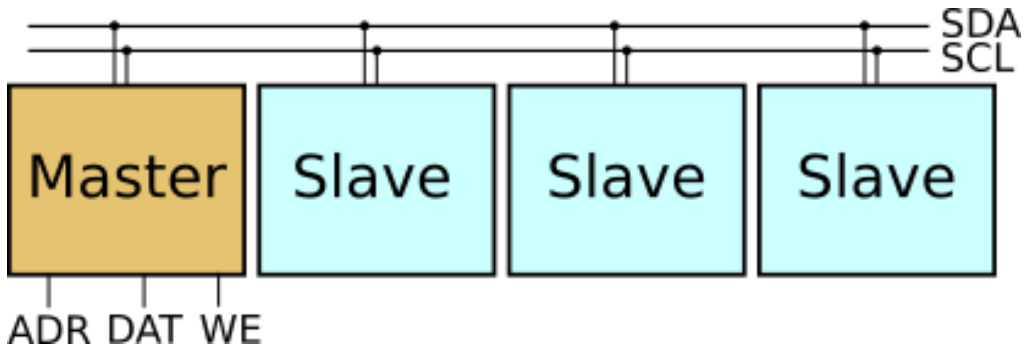
Many information-flow properties can be formalized as hyperproperties.

Noninterference as hyperproperty:

$$\{T \subseteq 2^{Traces} \mid \forall t, t' \in T : t =_{I_{public}} t' \Rightarrow t =_{O_{\text{public}}} t'\}$$

Case Study 1: Information flow in the I2C Bus



- Under which circumstances can information flow from the inputs through the Master to the bus (and vice versa)?
- How much information can flow?
- Is there an expiration date for information?

Case study 2: Symmetry in Protocols

while $(true)$ {
(1) choosing[i] = $true$;
(2) number[i] = max(number)+1;
(3) choosing[i] = $false$;
(4) **for** (int j=0; $j < n$; j++) {
(5) **while** (choosing[j]) { ; }
(6) **while** ($j \neq i \wedge$ number[j] $\neq 0 \wedge$ (number[j],j) $<$ (number[i],i)) { ; }
}
(7) **critical**
(8) number[i] = 0;
(9) **non-critical**
}

- Are the clients treated symmetrically?

Case study 3: Error-resistant codes

Different encoders from OpenCores.org.

- 8bit-10bit encoder, decoder
- Huffman encoder
- Hamming encoder

- Do codes for distinct inputs have at least Hamming distance d?

Automatic analysis techniques

- Security type systems
- Program analysis
- Dynamic approaches/taint tracking

Common problem: single-property techniques

Automatic analysis techniques

- Security type systems
- Program analysis
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Common problem: single-property techniques

This tutorial:
A unifying framework for the analysis of hyperproperties

Overview

I HyperLTL
II Examples
III Model Checking
IV Satisfiability
V Beyond HyperLTL

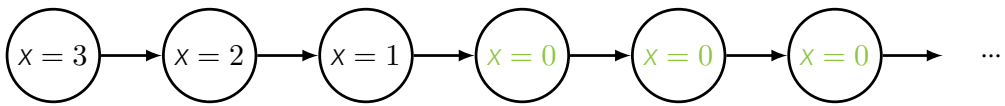
Section I

HyperLTL

Temporal logics

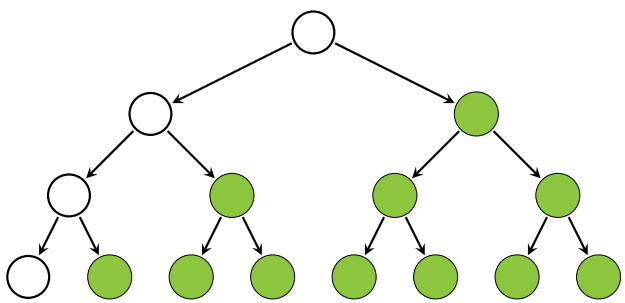
LTL: Specifies computations

Example: $\mathsf{FG}\ x = 0$ "from some point on x is 0"



CTL/CTL*: Specifies computation trees

Example: $\mathsf{AGEF}\ x = 0$ "x may always become 0 in the future"



Temporal logics

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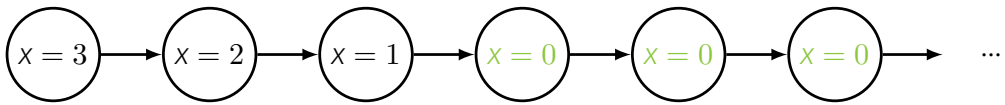

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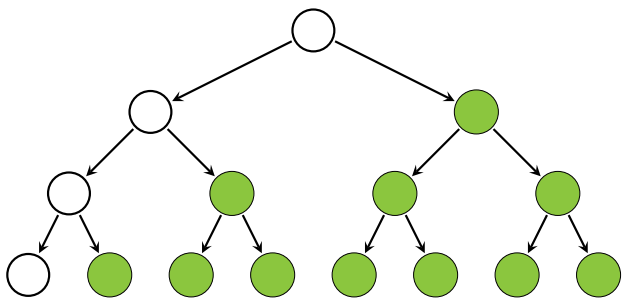
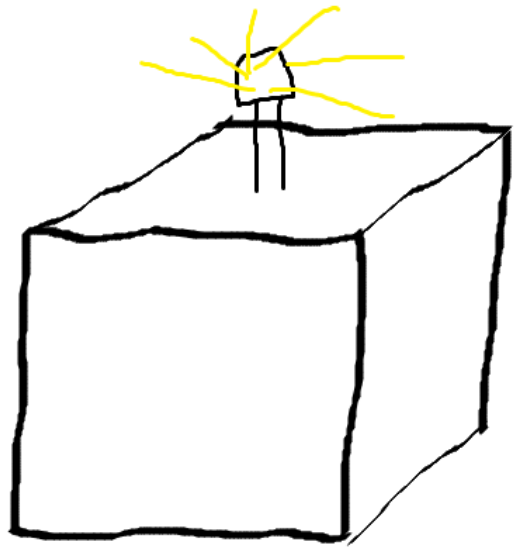


Suitable for specifying information-flow properties?

A Simple Information-flow Policy

"All executions have the light on at the same time."



"For all pairs of executions and all points in time, the light is on on the one execution iff it is on on the other execution."

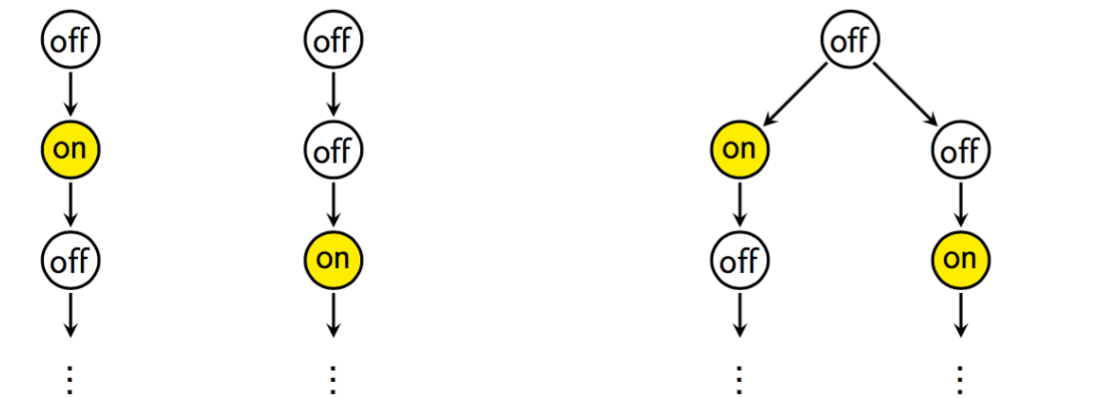
Information flow properties compare multiple executions!

LTL?

Syntax: $\varphi ::= a_\pi \mid \mathsf{X}\psi \mid \mathsf{G}\psi \mid \mathsf{F}\psi \mid \psi \mathsf{U} \psi \mid \psi W \psi$

Semantics: $K \models \varphi$ iff $Traces(K) \subseteq Traces(\varphi)$

"All executions have the light on at the same time."



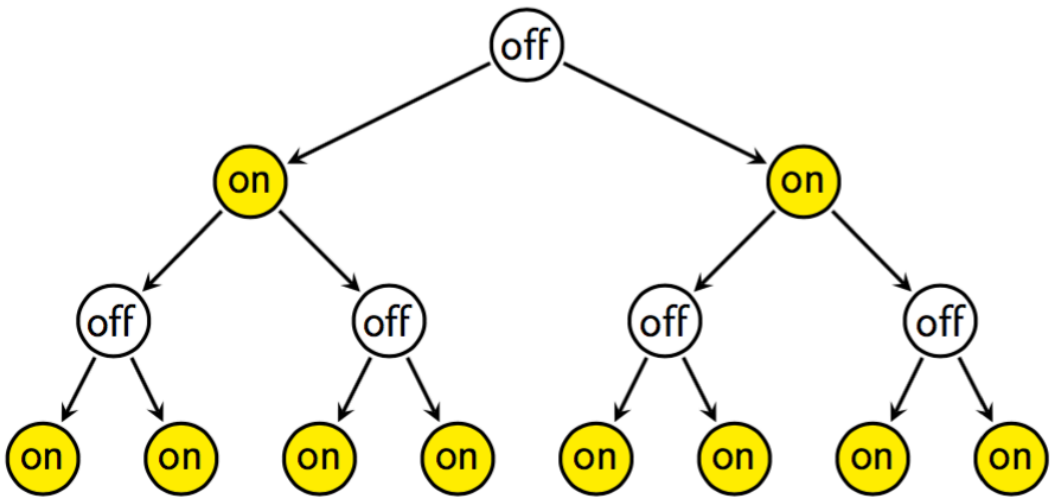
X on XX on (X on) ∨ (XX on) ?

CTL*?

Syntax: $\varphi ::= a \mid \mathsf{A}\varphi \mid \mathsf{E}\varphi \mid \mathsf{X}\varphi \mid \mathsf{G}\varphi \mid \varphi \mathsf{U} \varphi \mid \ldots$

Semantics: $K \models \mathsf{A}\varphi$ iff for all $p \in \mathsf{Paths}(K) : p \models \varphi$

"All executions have the light on at the same time." $\mathsf{AA}\varphi$ **?**



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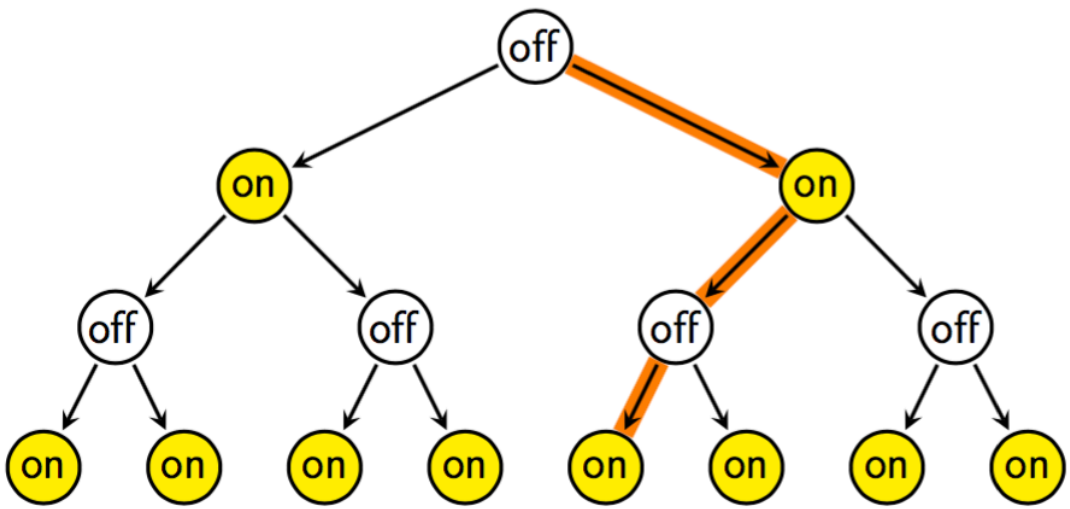

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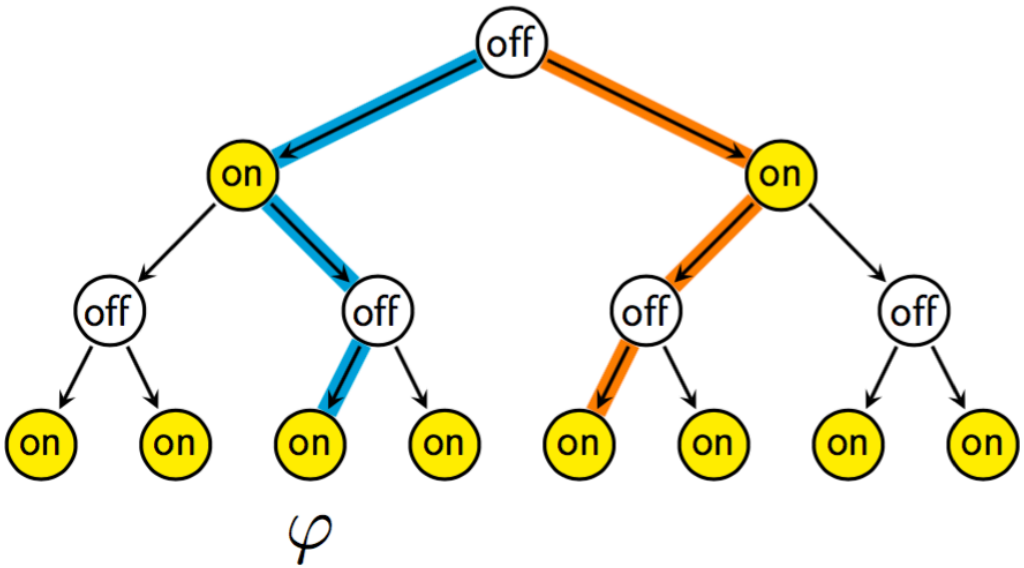

HyperLTL

Quantifiers with trace variables: $\forall \pi . \varphi \quad \exists \pi . \varphi$

Syntax: $\varphi ::= \forall \pi . \varphi \mid \exists \pi . \varphi \mid \psi$

$\psi ::= a_\pi \mid \mathsf{X}\psi \mid \mathsf{G}\psi \mid \mathsf{F}\psi \mid \psi \mathsf{U} \psi \mid \psi W \psi$

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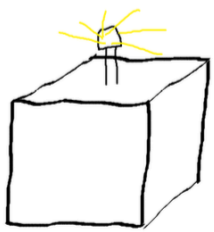
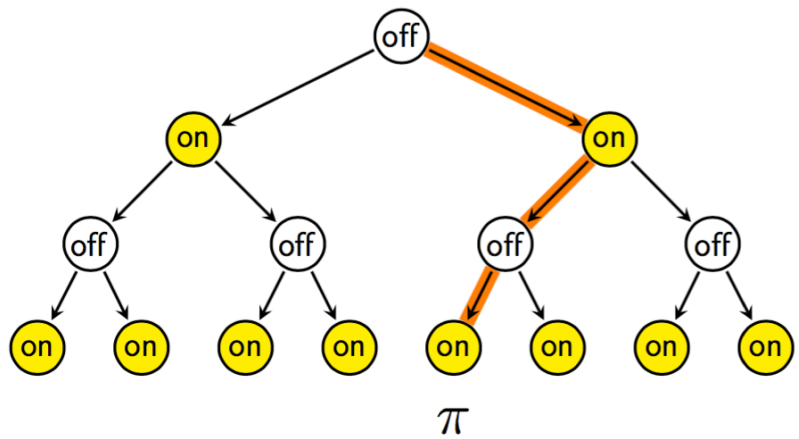
$$\forall \pi . \forall \pi' . \mathsf{G}(\mathsf{on}_\pi \leftrightarrow \mathsf{on}_{\pi'})$$





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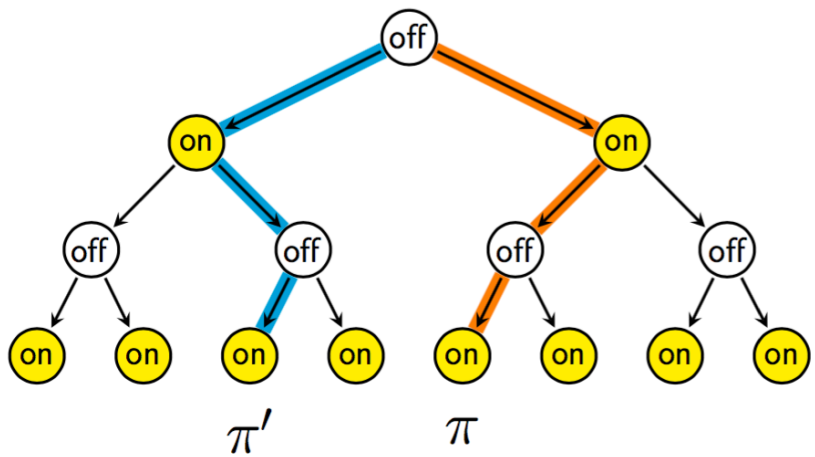

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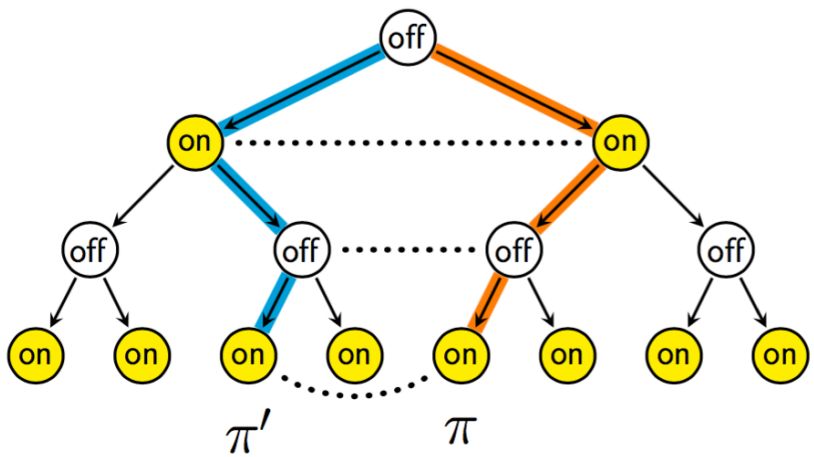

Semantics

$$
\begin{array}{lll}
\Pi \models_T a_\pi & \text{iff} & a \in \Pi(\pi)(0) \\
\Pi \models_T \mathsf{G}\varphi & \text{iff} & \forall i \geq 0 : \ \Pi[i, \infty] \models_T \varphi \\
\Pi \models_T \forall \pi.\ \varphi & \text{iff} & \forall t \in T : \ \Pi[\pi \mapsto t] \models_T \varphi
\end{array}
$$

Semantics given with respect to a set of traces T and a trace environment $\Pi : \mathit{Vars} \to T$

A Kripke structure K satisfies a HyperLTL formula φ iff $\emptyset \models_{\mathit{Traces}(K)} \varphi$

Semantics

$$
\begin{aligned}
\Pi \models_T a_\pi \quad & \text{iff} \quad a \in \Pi(\pi)(0) \\
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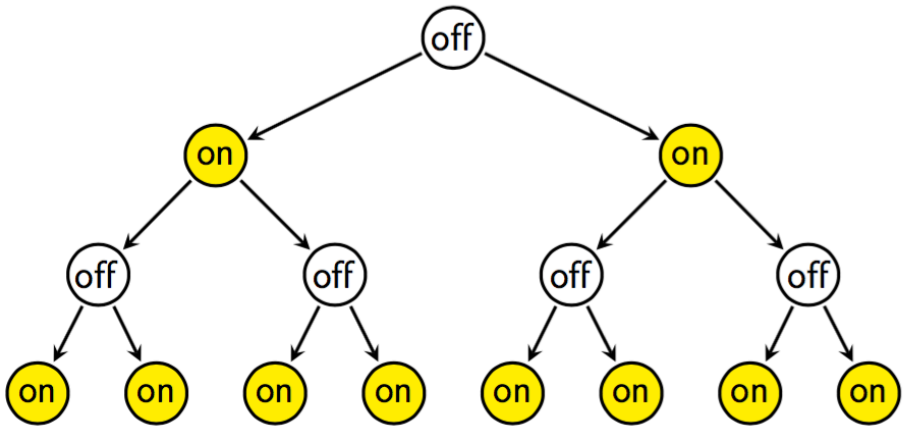

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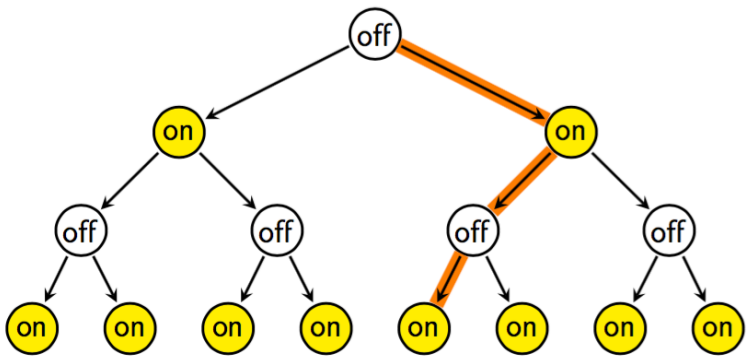

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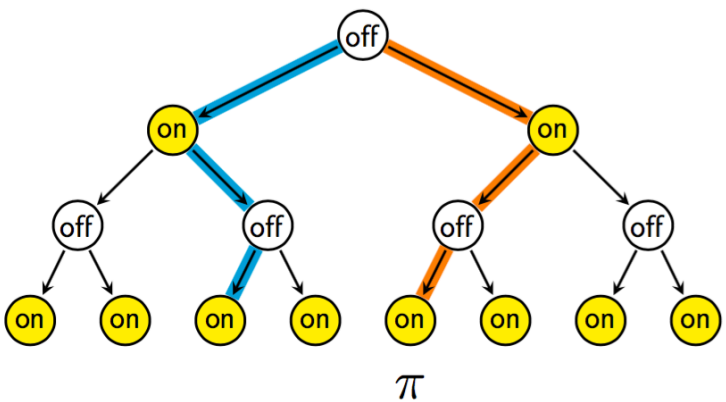

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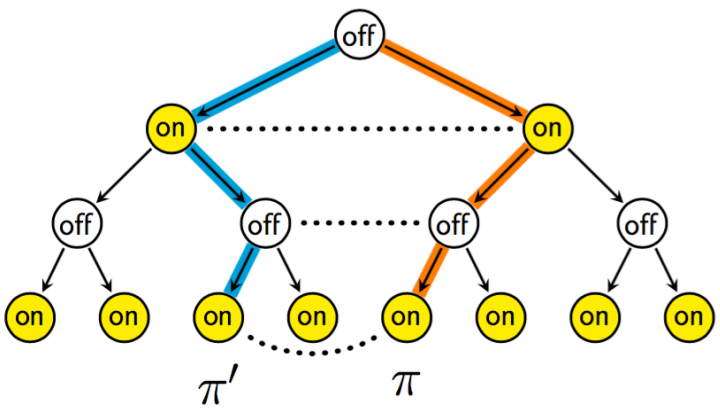

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3. $\{\pi \mapsto t[i, \infty], \pi' \mapsto t'[i, \infty]\} \models_T \mathsf{on}_\pi \leftrightarrow \mathsf{on}_{\pi'}$

Full Semantics

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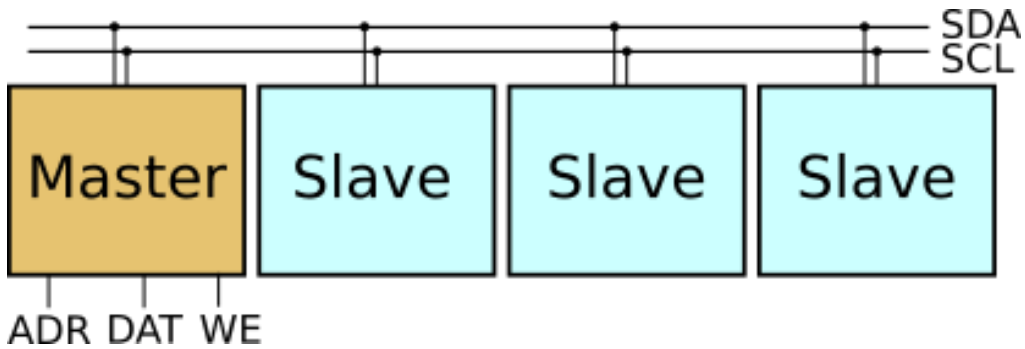
\Pi \models_T \neg\varphi & \text{iff} & \Pi \not\models_T \varphi \\
\Pi \models_T \varphi_1 \vee \varphi_2 & \text{iff} & \Pi \models_T \varphi_1 \text{ or } \Pi \models_T \varphi_2 \\
\Pi \models_T \mathsf{X}\varphi & \text{iff} & \Pi[1, \infty] \models_T \varphi \\
\Pi \models_T \mathsf{F}\varphi & \text{iff} & \exists i \geq 0 : \ \Pi[i, \infty] \models_T \varphi \\
\Pi \models_T \varphi_1 \,\mathsf{U}\, \varphi_2 & \text{iff} & \text{there exists } i \geq 0 : \Pi[i, \infty] \models_T \varphi_2 \\
 & & \quad \text{and for all } 0 \leq j < i \text{ we have } \Pi[j, \infty] \models_T \varphi_1 \\
\Pi \models_T \varphi_1 W \varphi_2 & \text{iff} & \Pi \models_T \varphi_1 \,\mathsf{U}\, \varphi_2 \text{ or } \Pi \models_T \mathsf{G}\varphi_1
\end{array}
$$

Section II

Examples

Case Study 1: Information flow in the I2C Bus



- Under which circumstances can information flow from the inputs through the Master to the bus (and vice versa)?

$$\forall\pi.\forall\pi'.\ \mathsf{G}(\overline{\mathsf{DAT}}_\pi = \overline{\mathsf{DAT}}_{\pi'}) \Rightarrow\ \mathsf{G}(\mathsf{SDA}_\pi = \mathsf{SDA}_{\pi'})$$

$P_\pi = P_{\pi'}$ is defined as $\bigwedge_{a\in P} a_\pi \leftrightarrow a_{\pi'}$.

$\overline{P}_\pi = \overline{P}_{\pi'}$ is defined as $(I \setminus P)_\pi = (I \setminus P)_{\pi'}$.

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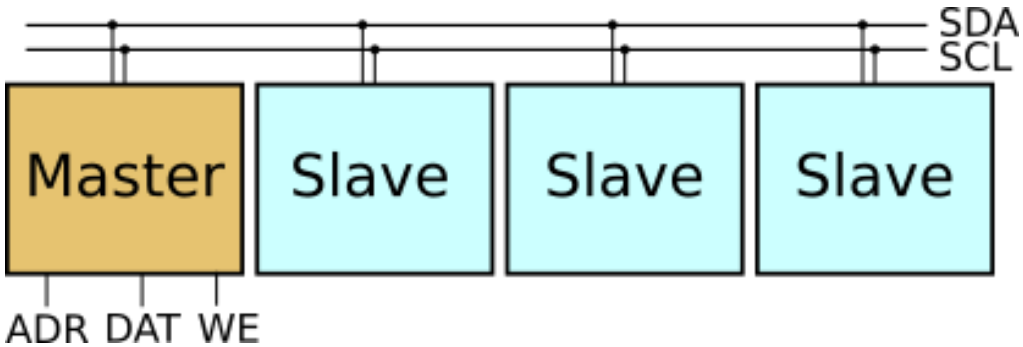

- Under which circumstances can information flow from the inputs through the Master to the bus (and vice versa)?

$$\forall\pi.\forall\pi'.\ \mathsf{G}\left(\neg \mathsf{WE} \wedge \overline{\mathsf{DAT}}_\pi = \overline{\mathsf{DAT}}_{\pi'}\right) \Rightarrow\ \mathsf{G}\left(\mathsf{SDA}_\pi = \mathsf{SDA}_{\pi'}\right)$$

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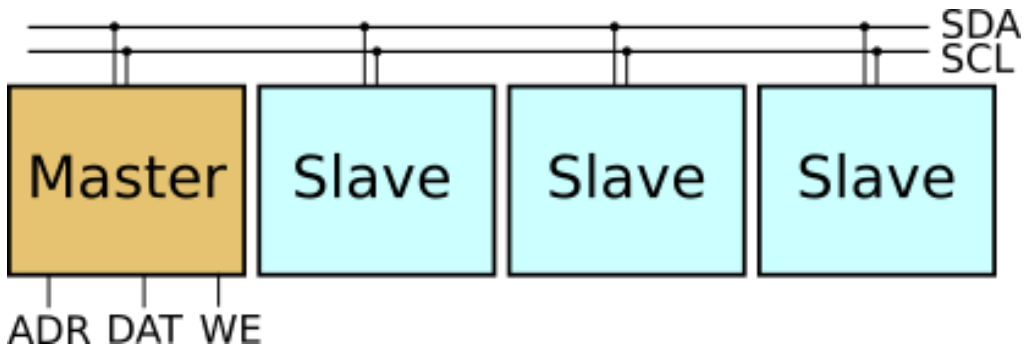

- How much information can flow?

$$\neg\exists\pi_0.\ \ldots\ .\exists\pi_{2^n}.\left(\bigwedge_i \mathsf{G}\ \overline{\mathsf{DAT}}_{\pi_0} = \overline{\mathsf{DAT}}_{\pi_i}\right) \wedge \bigwedge_{i\neq j} \mathsf{F}\ \mathsf{SDA}_\pi \neq \mathsf{SDA}_{\pi'}$$

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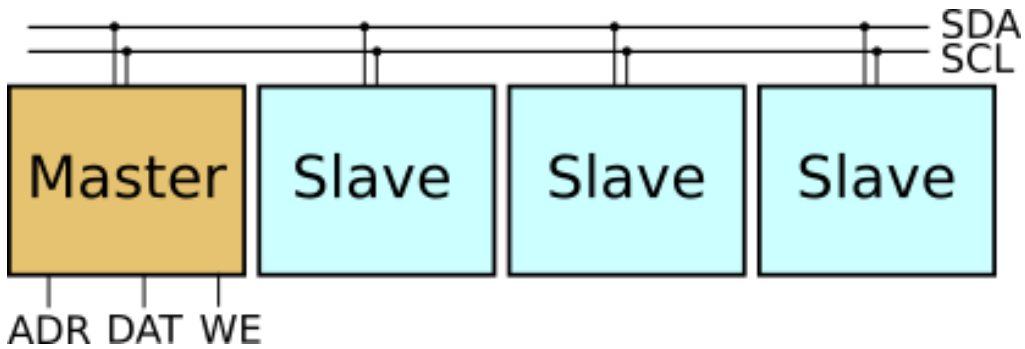

- Is there an expiration date for information?

$$\forall \pi . \forall \pi' . \left(\overline{\mathsf{DAT}}_{\pi} = \overline{\mathsf{DAT}}_{\pi'} \ \mathsf{U} \ \mathsf{G}\left(I_{\pi} = I_{\pi'}\right)\right) \Rightarrow \mathsf{F}\ \mathsf{G}\left(\mathsf{SDA}_{\pi} = \mathsf{SDA}_{\pi'}\right)$$

Variants of noninterference in HyperLTL

- Observational determinism [Zdancewich&Myers'03]

$$\forall \pi . \forall \pi' .\ lowIn_{\pi} = lowIn_{\pi'} \Rightarrow \mathsf{G}(lowOut_{\pi} = lowOut_{\pi'})$$

- Generalized noninterference [McCullough'88]

$$\forall \pi . \forall \pi' . \exists \pi'' .\ \mathsf{G}(highIn_{\pi} = highIn_{\pi''}) \wedge \mathsf{G}(lowIn_{\pi'} = lowIn_{\pi''} \ \wedge \ lowOut_{\pi'} = lowOut_{\pi''})$$

- Noninference [McLean'94]

$$\forall \pi . \exists \pi' .\ \mathsf{G}(highIn_{\pi'}) \wedge \mathsf{G}(lowIn_{\pi} = lowIn_{\pi'} \ \wedge \ lowOut_{\pi} = lowOut_{\pi'})$$

Case study 2: Symmetry in Protocols

while $(true)$ {
(1) choosing[i] = $true$;
(2) number[i] = max(number)+1;
(3) choosing[i] = $false$;
(4) **for** (int j=0; $j < n$; j++) {
(5) **while** (choosing[j]) { ; }
(6) **while** ($j \neq i \wedge$ number[j] $\neq 0 \wedge$ (number[j],j) $<$ (number[i],i)) { ; }
}
(7) **critical**
(8) number[i] = 0;
(9) **non-critical**
}

- Are the clients treated symmetrically?

$$\forall \pi . \forall \pi' . \, \mathsf{G}(select\_0_{\pi} \leftrightarrow select\_1_{\pi'} \ \wedge \ select\_1_{\pi} \leftrightarrow select\_0_{\pi'}) \Rightarrow \mathsf{G}(critical\_0_{\pi} \leftrightarrow critical\_1_{\pi'} \ \wedge \ critical\_1_{\pi} \leftrightarrow critical\_0_{\pi'})$$

Case study 2: Symmetry in Protocols

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(1) choosing[i] = $true$;
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(5) **while** (choosing[j]) { ; }
(6) **while** ($j \neq i \wedge$ number[j] $\neq 0 \wedge$ $\begin{array}{c} f \wedge (\text{number}[j], j) < (number[i], i) \\ \vee \\ \neg f \wedge (\text{number}[j], i) < (number[i], j) \end{array}$) { ; }
}
(7) **critical**
(8) number[i] = 0;
(9) **non-critical**
}

$$\forall \pi . \forall \pi' .\ \mathsf{G}(select\_0_\pi \leftrightarrow select\_1_{\pi'} \ \wedge\ select\_1_\pi \leftrightarrow select\_0_{\pi'} \ \wedge\ f_\pi \leftrightarrow \neg f_{\pi'}) \ \Rightarrow \mathsf{G}(critical\_0_\pi \leftrightarrow critical\_1_{\pi'} \ \wedge\ critical\_1_\pi \leftrightarrow critical\_0_{\pi'})$$

Case study 3: Error-resistant codes

Different encoders from OpenCores.org.

- 8bit-10bit encoder, decoder
- Huffman encoder
- Hamming encoder

- Do codes for distinct inputs have at least Hamming distance d?

$$\forall\pi.\forall\pi'.\ \mathsf{F}\,(\mathsf{DAT}_\pi \neq \mathsf{DAT}_{\pi'}) \ \Rightarrow\ \neg\mathsf{Ham}(d,\pi,\pi')$$

with $\mathsf{Ham}(0,\pi,\pi') = \text{false}$ and $\mathsf{Ham}(d,\pi,\pi')$ is:

$$o_\pi = o_{\pi'}\ \mathsf{W}\ \left(o_\pi \neq o_{\pi'}\ \wedge\ \mathsf{X}\ \mathsf{Ham}(d-1,\pi,\pi')\right)$$

Section III

Model Checking

Model Checking

- Reduction to emptiness of Büchi word automata
- Alphabet consists of tuples of states
- Projection and complementation handle quantifiers.

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Negated: $\exists\pi.\exists\pi'.\ \mathsf{G}(\mathsf{i}_\pi \leftrightarrow \mathsf{i}_{\pi'}) \ \wedge\ \mathsf{F}(\mathsf{o}_\pi \not\leftrightarrow \mathsf{o}_{\pi'})$

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Model Checking

Complexity depends on the quantifier alternation depth.

0. $\forall\pi.\forall\pi'.\psi$ PSPACE in $|\psi|$, NLOGSPACE in $|K|$
   - Observational determinism

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   - Noninference
   - Generalized noninterference
2. ...

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0. $\forall\pi.\forall\pi'.\psi$ PSPACE in $|\psi|$, NLOGSPACE in $|K|$
   - Observational determinism
1. $\forall\pi.\exists\pi'.\psi$ EXPSPACE in $|\psi|$, PSPACE in $|K|$
   - Noninference
   - Generalized noninterference
2. ...

Rarely need more than one quantifier alternation!

Symbolic Model Checking for Circuits

- Alternation-free HyperLTL (and HyperCTL*)
- Clean extension of the circuit construction for LTL
- Leverages existing symbolic model checkers (e.g. ABC)

A Circuit Based Approach

$$\forall\pi.\forall\pi'.\ \mathsf{G}(\mathsf{i}_\pi \leftrightarrow \mathsf{i}_{\pi'}) \ \Rightarrow\ \mathsf{G}(\mathsf{o}_\pi \leftrightarrow \mathsf{o}_{\pi'})$$

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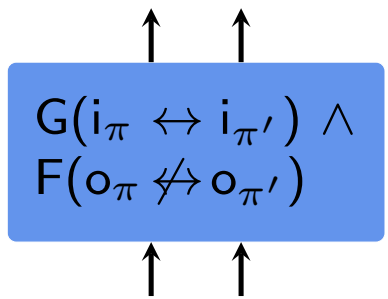

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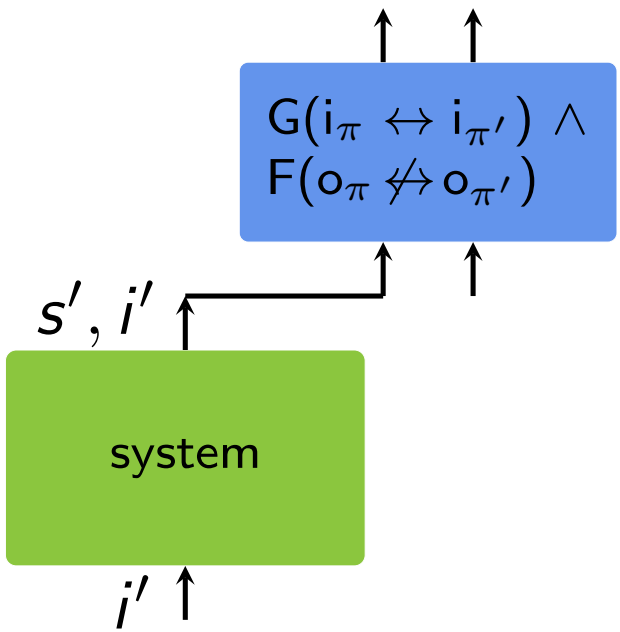

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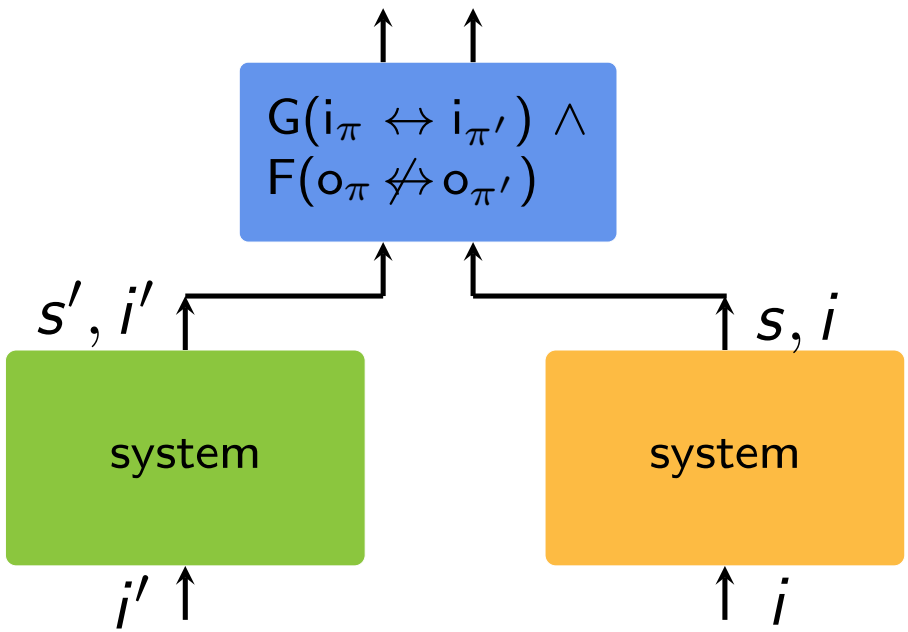


Implementation - MCHyper

A transformation on Aiger circuits

Workflow

1. Convert VHDL/Verilog to Aiger
2. Run MCHyper with a formula and the circuit
3. Call a hardware model checker on the resulting circuit

Tool website:
`https://www.react.uni-saarland.de/tools/mchyper/`

Information flow - Experimental Data

					Verification time in s			
		Model	#Latches	#Gates	IC3	INT	BMC	Result
IF1	(NI1)	I2C Master	254	1207	95.17	1.13	0.07	×
IF2	(NI2)				53.08	1.16	0.08	×
IF3	(NI3)				168.96	1.38	-	✓
IF4	(NI4)				438.41	1.01	0.09	×
IF5	(NI5)				717.74	8.31	0.77	×
IF6	(NI6)				186.20	1.10	0.07	×
IF7	(NI7)				TO	6.82	0.55	×
IF8	(NI8)				1557.14	2.92	0.16	×
IF9	(NI2′)	Ethernet	21093	70837	TO	155.77	6.27	×

Symmetry in Protocols - Experimental Data

					Verification time in s			
		Model	#Latches	#Gates	IC3	INT	BMC	Result
Sym1	(S1)	Bakery	46	1829	6.34	0.88	0.08	×
Sym2	(S2)				168.59	464.52	7.00	×
Sym3		Bakery.a	47	1588	69.12	TO	71.92	×
Sym4	(S3)	Bakery.a.n	47	1618	26.31	4.75	0.39	×
Sym5		Bakery.a.n.s	47	1532	66.41	TO	-	✓
Sym6	(S4)				16.83	TO	-	✓
Sym7	(S5)	Bakery.a.n.s.5proc	90	3762	97.45	TO	-	✓
Sym8	(S6)				13.59	TO	-	✓
Sym9	(S7)	Bakery.a.n.s.7proc	136	6775	312.53*	TO	-	✓

Error Correcting Codes - Experimental Data

					Verification time in s		
		Model	#Latches	#Gates	IC3	INT	BMC	Result
Huff1	(HD1)	Huffman_enc	19	571	3.08	37.19	-	✓
Huff2	(HD2)				0.62	0.09	0.02	×
8b10b_1	(HD1)	8b10b_enc	39	271	0.32	0.09	0.02	×
8b10b_2	(HD1′)				1.19	9.06	-	✓
8b10b_3	(HD2′)				0.03	0.04	0.02	×
8b10b_4	(HD1″)	8b10b_dec	19	157	0.05	0.09	-	✓
Hamm1	($HD1_1$)	Hamming_enc	27	47	0.02	0.04	0.02	×
Hamm2	($HD1_2$)				0.02	0.03	0.02	×
Hamm3	($HD1_3$)				0.03	0.04	0.02	×
Hamm3'	($HD1'_3$)				7.34	0.18	-	✓
Hamm4	($HD1_4$)				66.93	0.10	-	✓
Hamm5	($HD2_1$)				11.83	1.31	-	✓
Hamm6	($HD2_2$)				14.44	0.78	-	✓
Hamm7	(HD3)				12.23	1.25	-	✓

Section IV

Satisfiability

Satisfiability of HyperLTL

HyperLTL-SAT is the problem to decide whether there exists a non-empty trace set T satisfying a HyperLTL formula φ.

Application: Two versions of Observational Determinism:

- $\forall \pi . \forall \pi' . \mathsf{G}(I_\pi = I_{\pi'}) \rightarrow \mathsf{G}(O_\pi = O_{\pi'})$
- $\forall \pi . \forall \pi' . (O_\pi = O_{\pi'}) \,\mathsf{W}\, (I_\pi \neq I_{\pi'})$

Which version is stronger?

Challenge

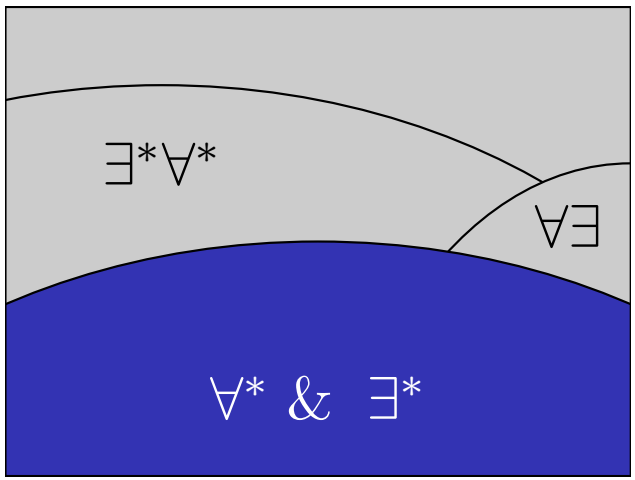
LTL Satisfiability Solving

- Translate LTL formula into Büchi automaton
- Check the automaton for emptiness
- PSPACE-complete

HyperLTL Satisfiability Solving

- A Hyperproperty is not necessarily ω-regular
- Standard automata approach cannot be applied

Solving HyperLTL-SAT



1. Alternation-free fragments ($\forall^* \,\&\, \exists^*$)
2. Alternation starting with existential quantifier ($\exists^*\forall^*$)
3. Alternation starting with universal quantifier ($\forall^*\exists^*$)

Existential Fragment

Theorem

$\exists^*$ *HyperLTL-SAT is PSPACE-complete.*

Example

$\exists \pi_0 \exists \pi_1 .\ \mathsf{G}\, a_{\pi_0} \wedge \mathsf{G}\, b_{\pi_0} \wedge \mathsf{G}\, c_{\pi_0} \wedge \mathsf{G}\, a_{\pi_1} \wedge \mathsf{G} \neg c_{\pi_1}$

Replace indexed atomic propositions with fresh atomic propositions.

$$\mathsf{G}\, a_0 \wedge \mathsf{G}\, b_0 \wedge \mathsf{G}\, c_0 \wedge \mathsf{G}\, a_1 \wedge \mathsf{G} \neg c_1$$

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$$\mathsf{G}\, a_0 \wedge \mathsf{G}\, b_0 \wedge \mathsf{G}\, c_0 \wedge \mathsf{G}\, a_1 \wedge \mathsf{G} \neg c_1$$
$$t : \{a_0, b_0, c_0, a_1\}^\omega$$

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Replace indexed atomic propositions with fresh atomic propositions.

$$\mathsf{G}\, a_0 \wedge \mathsf{G}\, b_0 \wedge \mathsf{G}\, c_0 \wedge \mathsf{G}\, a_1 \wedge \mathsf{G} \neg c_1$$
$$t : \{a_0, b_0, c_0, a_1\}^\omega$$

$$T = \{\{a, b, c\}^\omega, \{a\}^\omega\}$$

Universal Fragment

Theorem

$\forall^*$ *HyperLTL-SAT is PSPACE-complete.*

Example

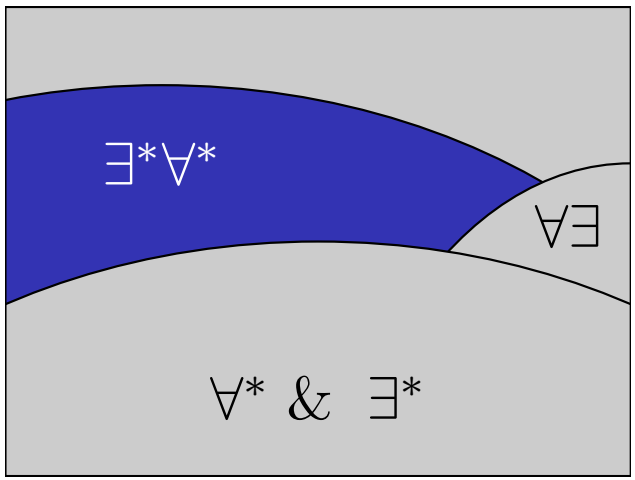
$$\forall\pi\forall\pi'.\mathsf{G}\,b_\pi \wedge \mathsf{G}\neg b_{\pi'} \quad \equiv \quad \mathsf{G}\,b \wedge \mathsf{G}\neg b$$

$$\Downarrow\ \Downarrow \qquad\qquad\qquad\qquad \Downarrow$$

$$t \quad t \qquad\qquad\qquad\qquad \text{unsatisfiable}$$

Discard indexes from indexed propositions

Solving HyperLTL-SAT



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2. Alternation starting with existential quantifier ($\exists^*\forall^*$)
3. Alternation starting with universal quantifier ($\forall^*\exists^*$)

$\exists^*\forall^*$ HyperLTL-SAT

Lemma

For every $\exists\pi_1 \ldots \exists\pi_n \forall\pi'_1 \ldots \forall\pi'_m.\varphi$ HyperLTL formula, there exists an equisatisfiable $\exists^*$ HyperLTL formula.

Example

$\exists\pi_0\exists\pi_1\forall\pi'_0\forall\pi'_1.\ (\mathsf{G}\, a_{\pi'_0} \wedge \mathsf{G}\, b_{\pi'_1}) \wedge (\mathsf{G}\, c_{\pi_0} \wedge \mathsf{G}\, d_{\pi_1})$

Unroll universal quantifiers

$$\begin{aligned}\exists\pi_0\exists\pi_1.\ & (\mathsf{G}\, a_{\pi_0} \wedge \mathsf{G}\, b_{\pi_0}) \wedge (\mathsf{G}\, c_{\pi_0} \wedge \mathsf{G}\, d_{\pi_1}) \\ & \wedge (\mathsf{G}\, a_{\pi_1} \wedge \mathsf{G}\, b_{\pi_0}) \wedge (\mathsf{G}\, c_{\pi_0} \wedge \mathsf{G}\, d_{\pi_1}) \\ & \wedge (\mathsf{G}\, a_{\pi_0} \wedge \mathsf{G}\, b_{\pi_1}) \wedge (\mathsf{G}\, c_{\pi_0} \wedge \mathsf{G}\, d_{\pi_1}) \\ & \wedge (\mathsf{G}\, a_{\pi_1} \wedge \mathsf{G}\, b_{\pi_1}) \wedge (\mathsf{G}\, c_{\pi_0} \wedge \mathsf{G}\, d_{\pi_1})\end{aligned}$$

Complexity of $\exists^*\forall^*$ HyperLTL-SAT

Theorem
Let n be the number of existential quantifiers and m be the number of universal quantifiers. $\exists^*\forall^*$ HyperLTL-SAT is EXPSPACE-complete in m.

- Unrolling results in formula of size $O(n^m)$.
- Hardness follows from an encoding of an EXPSPACE-bounded Turing machine in this fragment.

Application: Implication Checking of Quantifier-alternation-free Hyperproperties

ψ implies φ?
Check the negation $\neg(\psi \rightarrow \varphi)$ for unsatisfiability.

Example

To determine whether the HyperLTL formula $\forall\pi_0 \ldots \forall\pi_n.\ \psi$ implies the HyperLTL formula $\forall\pi'_0 \ldots \forall\pi'_m.\ \varphi$, we check the $\exists^n\forall^m$ formula $\exists\pi_1 \ldots \pi_n \forall\pi'_0 \ldots \pi'_m.\ \psi \wedge \neg\varphi$ for unsatisfiability.

Theorem

Implication Checking of quantifier-alternation-free HyperLTL formulas is EXPSPACE-complete.

$\exists^*\forall^b$ HyperLTL-SAT

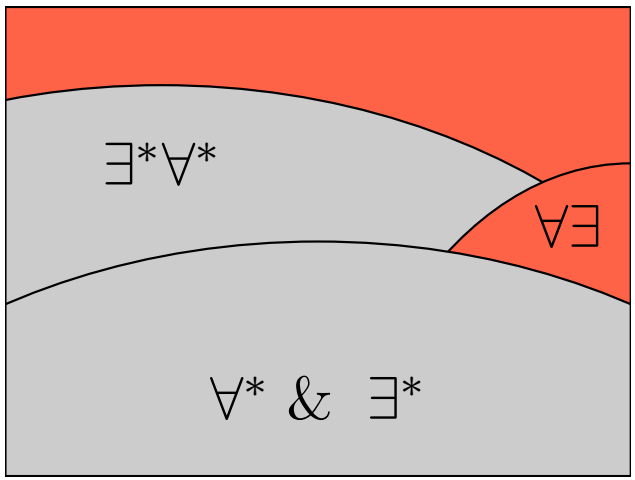
Theorem

Bounded $\exists^*\forall^b$ HyperLTL-SAT is PSPACE-complete.

Observation: In practice, many properties of interest quantify universally over pairs of traces

$$\forall\pi.\forall\pi'.\mathsf{G}(I_\pi = I_{\pi'}) \rightarrow \mathsf{G}(O_\pi = O_{\pi'})$$

Solving HyperLTL-SAT



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The Power of ∀∃

$$\forall \pi \exists \pi'.\ a_{\pi'} \tag{1}$$
$$\wedge\ \mathsf{G}(a_\pi \rightarrow \mathsf{X}\mathsf{G}\neg a_\pi) \tag{2}$$
$$\wedge\ \mathsf{G}(a_\pi \rightarrow \mathsf{X} a_{\pi'}) \tag{3}$$

$$t_1 :\ \{a\}(\{\})^\omega$$
$$t_2 :\ \{\}\{a\}(\{\})^\omega$$
$$t_3 :\ \{\}\{\}\{a\}(\{\})^\omega$$
$$\ldots$$

→ Model has infinitely many traces.

Undecidability of ∀∃ HyperLTL-SAT

Theorem
The satisfiability problem for any fragment of HyperLTL that contains the ∀∃ formulas is undecidable.

- Undecidability follows from a reduction from Post's Correspondence Problem.

Summary HyperLTL-SAT

$\exists^*$	$\forall^*$	$\exists^*\forall^*$	Bounded $\exists^*\forall^*$	$\forall\exists$
PSpace-complete	PSpace-complete	EXPSpace-complete	PSpace-complete	undecidable

Section V

Beyond HyperLTL

Hyperproperties and branching-time logics

Observation:

Hyperproperties induce trace equivalence.

$$\forall K, K'.\ \ \mathsf{Traces}(K) = \mathsf{Traces}(K') \implies K \models H \leftrightarrow K' \models H$$

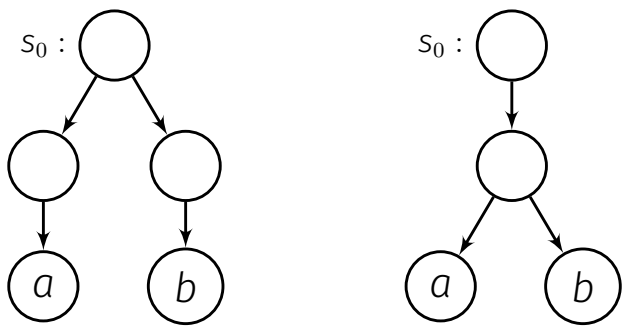
Hyperproperties and branching-time logics

Observation:

Hyperproperties induce trace equivalence.

$$\forall K, K'.\ \ \mathsf{Traces}(K) = \mathsf{Traces}(K') \implies K \models H \leftrightarrow K' \models H$$

Hyperproperties are not models for branching-time logics.



HyperCTL*

Syntax: $\varphi ::= a_\pi \mid \forall\pi.\varphi \mid \exists\pi.\varphi \mid \mathsf{X}\varphi \mid \mathsf{G}\varphi \mid \varphi\,\mathsf{U}\,\varphi \mid \ldots$

- HyperLTL: no quantifiers under temporal operators
- HyperCTL*: no restriction
- HyperCTL* with 1 path variable $\approx$ CTL*

What do we get beyond HyperLTL and CTL*?

```
bool y;
bool x = read(); // secret
output(y);
```

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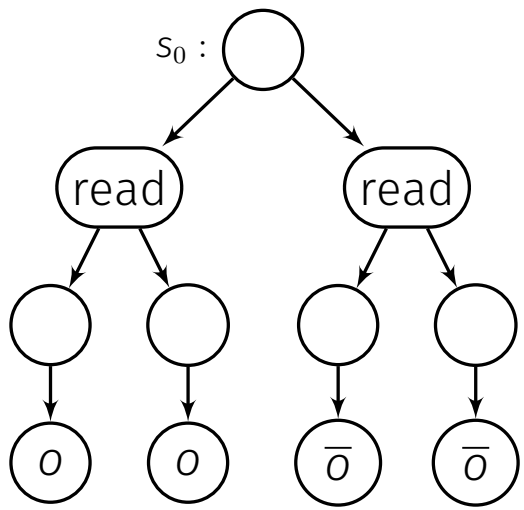

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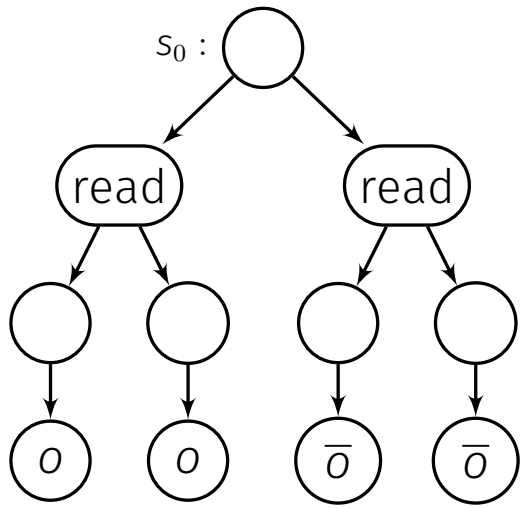
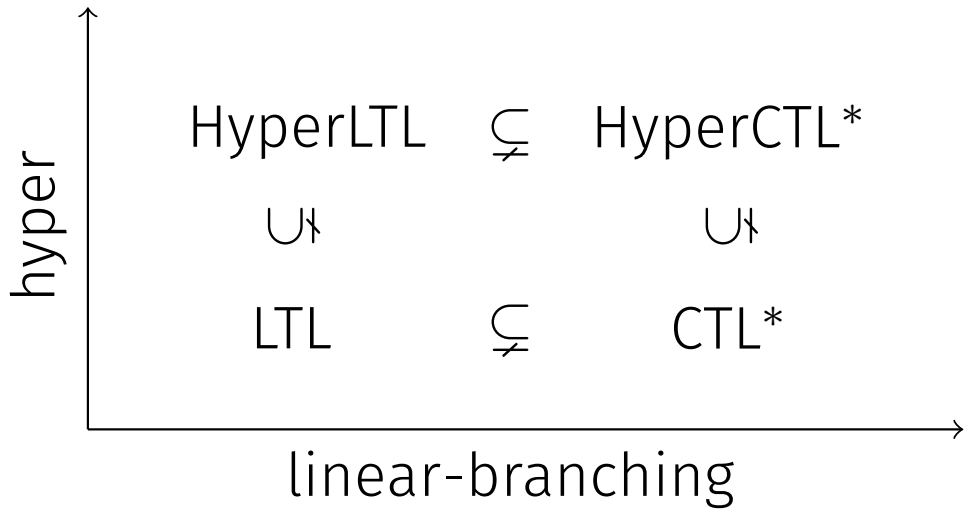


$$\forall \pi.\, \mathsf{G}(\mathsf{read}_\pi \rightarrow \forall \pi'.\, \mathsf{G}(o_\pi \leftrightarrow o_{\pi'}))$$

The Linear-Hyper-Branching Spectrum



- The induced process equivalence of HyperLTL is trace equivalence.

  Two systems with the same set of traces satisfy the same HyperLTL formulas.

- The induced process equivalence of HyperCTL* is bisimulation.

  Two bisimular systems satisfy the same HyperCTL formulas.*

Section VI

Conclusions

Conclusions

- HyperLTL is a versatile tool for information security and beyond.
- Model checking is decidable but, in general, hard.
- Satisfiability is in general undecidable, but efficient for alternation-free formulas.
- Many application areas
  - Information-flow control
  - Symmetries in distributed systems
  - Error resistant codes

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